Physics 481: Solid State Physics - Homework Solutions 10

Problem 1

a) $$\frac{C}{V} = \gamma T + \beta T^3$$

phonon part from Debye model $\frac{12}{5}\pi^4 \frac{N}{V} k_B \left(\frac{T}{\Theta_D}\right)^3$

electronic part from Fermi gas $\frac{\pi^2}{2} \frac{N}{V} k_B \left(\frac{T}{T_F}\right)$

the two parts become equal for

$$\frac{12}{5}\pi^4 \left(\frac{T_x}{\Theta_D}\right)^3 = \frac{\pi^2}{2}\left(\frac{T_x}{T_F}\right)$$

$$T_x^2 = \frac{\Theta_D^3}{T_F} \cdot \frac{5}{24\pi^2}$$

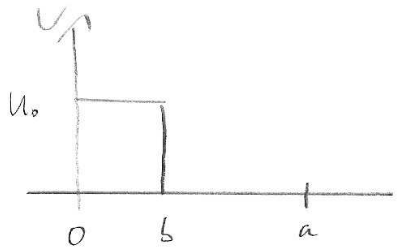
$$\boxed{T_x = \Theta_D \sqrt{\frac{\Theta_D}{T_F} \cdot \frac{5}{24\pi^2}}}$$

b) realistic values $\Theta_D \approx 300\ K$

$T_F \approx 30\,000\ K$

$$T_x \sim 5K$$

Problem 2



no i ($E<U_0$)

$$\psi = A_1 e^{q_1 x} + B_1 e^{-q_1 x} \quad (0<x<b)$$

$$\psi = A_2 e^{iq_2 x} + B_2 e^{-iq_2 x} \quad (b<x<a)$$

$$\frac{\hbar^2 q_1^2}{2m} = U_0 - E \ , \quad \frac{\hbar^2 q_2^2}{2m} = E$$

boundary conditions

$x=b$:
$$A_1 e^{q_1 b} + B_1 e^{-q_1 b} = A_2 e^{iq_2 b} + B_2 e^{-iq_2 b}$$
$$q_1 A_1 e^{q_1 b} - q_1 B_1 e^{-q_1 b} = iq_2 A_2 e^{iq_2 b} - iq_2 B_2 e^{-iq_2 b}$$

$x=a$
$$A_2 e^{iq_2 a} + B_2 e^{-iq_2 a} = (A_1 + B_1)\, e^{ika}$$
$$iq_2 A_2 e^{iq_2 a} - iq_2 B_2 e^{-iq_2 a} = (q_1 A_1 - q_1 B_1)\, e^{ika}$$

coefficient determinant must vanish

$$\begin{vmatrix} e^{q_1 b} & e^{-q_1 b} & -e^{iq_2 b} & -e^{-iq_2 b} \\ q_1 e^{q_1 b} & -q_1 e^{-q_1 b} & -iq_2 e^{iq_2 b} & +iq_2 e^{-iq_2 b} \\ e^{ika} & e^{ika} & -e^{iq_2 a} & -e^{-iq_2 a} \\ q_1 e^{ika} & -q_1 e^{ika} & -iq_2 e^{iq_2 a} & +iq_2 e^{-iq_2 a} \end{vmatrix} = 0$$

use Maple for determinant

$$0 = \cos ka - \Big(\frac{1}{4}\, e^{q_1 b + i q_2 b - i q_2 a} + e^{q_1 b - i q_2 b + i q_2 a} + e^{-q_1 b + i q_2 b - i q_2 a} + e^{-q_1 b + i q_2 a - i q_1 b}\Big)$$

$$- \frac{i(q_1^2 - q_2^2)}{8 q_1 q_2}\Big(e^{q_1 b + i q_2 a - i q_2 b} - e^{q_1 b + i q_2 b - i q_2 a} + e^{-q_1 b + i q_2 b - i q_2 a} - e^{-q_1 b + i q_2 a - i q_2 b}\Big)$$

$$\cos ka = \cosh(q_1 b)\cos\big(q_2(a-b)\big) + \frac{q_1^2 - q_2^2}{2 q_1 q_2}\sinh(q_1 b)\sin q_2(a-b)$$

c) $b \to 0$, $U_0 \to \infty$ $\qquad U_0 b \to \frac{W_0 \hbar^2}{ma}$

$$q_1 b \to 0 \qquad (q_1 \sim \sqrt{U_0})$$

$$\frac{q_1^2 - q^2}{2 q_1 q_2}\sin(q_1 b) \to \frac{q_1^2 - q_2^2}{2 q_2}\, b \to \frac{2 m U_0}{\hbar^2}\frac{b}{2 q_2}$$

$$\to \frac{W_0}{a q_2}$$

$$\boxed{\cos ka = \cos(q_2 a) + \frac{W_0}{a q_2}\sin q_2 a}$$

d) plots can be produced by calculating k as function of $\varepsilon = \frac{\hbar^2 q_2^2}{2m}$