Physics 481: Condensed Matter Physics - Homework Solutions 11

Microscopic Drude theory

a)

$$p = \left(1 - \frac{dt}{\tau}\right)^{\frac{t}{dt}} = e^{+\ln\left(1 - \frac{dt}{\tau}\right)\frac{t}{dt}}$$

Expand

$$p = e^{-t/\tau}$$

works forward and backward in time

b) assume collision at t_1

probability for next collision between

$t_2 = t_1 + t$ and $t_2 + dt$ is

$$p = \underbrace{e^{-t/\tau}}_{\text{no collision between } t_1 \text{ and } t_2} \; \frac{dt}{\tau}$$

collision between t_2 and $t_2 + dt$

c) Average over all electrons (from a)

$$t_f = t_D = \int_0^\infty \frac{dt}{\tau}\, t\, e^{-t/\tau} = \tau$$

d) Average over all collisions from b)

$$t_c = \int_0^\infty \frac{dt}{\tau}\, e^{-t/\tau} = \tau$$

e) <u>Bonus:</u>

probability distribution of $T = t_f + t_B$

$$W(T) = \frac{1}{\tau}\int_0^\infty dt_f \; e^{-t_f/\tau} \; \frac{1}{\tau}\int_0^\infty t_B \; e^{-t_B/\tau} \qquad \delta\left(T-(t_f+t_B)\right)$$

$$t_B = T - t_f$$

$$W(T) = \frac{1}{\tau^2}\int_0^T dt_f \; e^{-t_f/\tau} \qquad e^{-(T-t_f)/\tau}$$

$$W(T) = \frac{T}{\tau^2}\, e^{-T/\tau} \qquad \int dT\, W(T) = 1$$

$$\int dT\, T\, W(T) = 2\tau$$

$\Rightarrow$ factor T suppresses small distances

The difference between this result and d) is that e) averages over all electrons while d) average over all collisions. If an e^- has many collisions (short t) it contributes once in e) but many times in d)

Problem 2

a) • velocity after first collision $\vec{v}_1 = v_0 \hat{n}$
where $\hat{n}$ is a random unit vector

• velocity at second collision $\vec{v}_2 = v_0 \hat{n} + (-e) E t \hat{k}$
(E-field in z-direction)

$$\Delta E = \frac{1}{2m}\left(\left|v_0 \hat{n} + (-e) E t \hat{k}\right|^2 - v_0^2 \right)$$

$$= \frac{1}{2m}\left(2 v_0 \hat{n} \cdot (-e) E t \hat{k} + e^2 E^2 t^2 \right)$$

$$\langle \Delta E \rangle = \frac{1}{2m} e^2 E^2 t^2$$

averaged over $\hat{n}$, first term vanishes

b)

$$[\Delta E] = \int_0^\infty dt \, \frac{1}{\tau} e^{-t/\tau} \, \frac{1}{2m} e^2 E^2 t^2$$

$$= \frac{1}{2m} e^2 E^2 \tau^2 \underbrace{\int_0^\infty d\left(\frac{t}{\tau}\right) e^{-t/\tau} \left(\frac{t}{\tau}\right)^2}_{2} = \frac{e^2 E^2 \tau^2}{m}$$

energy loss per volume per time

$$\frac{\Delta Q}{V \Delta t} = [\Delta E] \cdot n \cdot \frac{1}{\tau} = \frac{n e^2 E^2 \tau}{m} = \sigma E^2$$

Wire $\quad V = L \cdot A \,, \quad R = \frac{1}{\sigma} \frac{L}{A} \,, \quad I = jA = \sigma E A$

$$\frac{\Delta Q}{\Delta t} = L \cdot A \, \frac{I^2}{\sigma A^2} = I^2 R$$

Semiclassical dynamics of tight-binding model

a) ansatz $|\psi\rangle = \sum_{xy} e^{i(k_x x + k_y y)} |xy\rangle$

$$H|\psi\rangle = \sum_{xy} U e^{i(k_x x + k_y y)} |xy\rangle$$

$$+ \sum_{xy} t \left\{ e^{i(k_x x + k_y y)} \left(|x+a, y\rangle + |x-a, y\rangle + |x, y+a\rangle + |x, y-a\rangle \right) \right\}$$

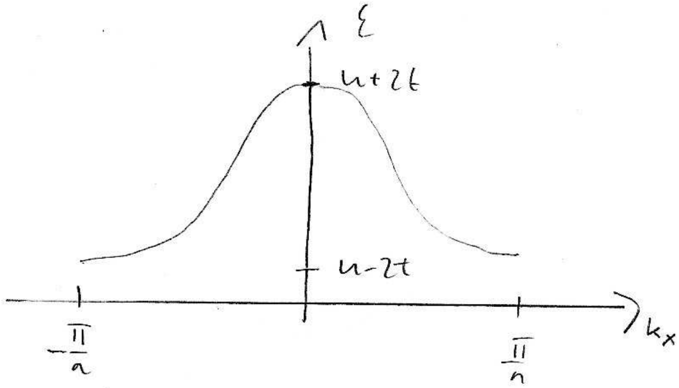
shift x, y in 2nd term

$$H|\psi\rangle = \sum_{xy} e^{i(k_x x + k_y y)} |xy\rangle$$

$$\left(U + t (e^{-ik_x a} + e^{ik_x a} + e^{-ik_y a} + e^{ik_y a} \right)$$

$$= |\psi\rangle \left(U + 2t \cos k_x a + 2t \cos k_y a \right)$$

$$\boxed{\varepsilon(\vec{k}) = U + 2t \cos k_x a + 2t \cos k_y a}$$



b) effective mass

$$M^{-1} = \frac{1}{\hbar^2}\frac{\partial^2 \varepsilon}{\partial k_\alpha \partial k_\beta}$$

$$M^{-1} = \frac{1}{\hbar^2}\begin{pmatrix} -2ta^2\cos k_x a & 0 \\ 0 & -2ta^2 \cos k_y a \end{pmatrix}$$

$$M_{xx} = -\frac{\hbar^2}{2ta^2\cos(k_x a)} \qquad M_{yy} = -\frac{\hbar^2}{2ta^2\cos(k_y a)}$$

$M \sim \frac{1}{t}$ makes sense since $t \sim E_{kin}$
(c.f. free e^-)

$M_{xx}, M_{yy} < 0$ at $\vec{k} = 0$ (see picture)

> 0 at $k_x, k_y = \frac{\pi}{a}$

c) 3D

$$\varepsilon(\vec{k}) = u + 2t(\cos k_x a + \cos k_y a + \cos k_z a)$$

no other change

Problem 4

a) $\omega = \frac{a\,e\,E}{\hbar}$ $\quad T = \frac{2\pi}{\omega} = \frac{2\pi\hbar}{a\,e\,E}$

$$E = \frac{2\pi\hbar}{aeT} = 6 \times 10^{7}\ \frac{V}{m}$$

b) Zener tunneling when

$$\frac{1}{\hbar}\frac{E_g}{eE}\sqrt{2mE_g} \approx 1$$

$$E = \frac{1}{\hbar}\frac{\sqrt{2m}\,E_g^{3/2}}{e} = 1.4 \times 10^{10}\ \frac{V}{m}$$

c) $E = \frac{2\pi\hbar}{aeT} \approx 1380\ \frac{V}{m}$ $\quad$ feasible