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Problem 4.1

a) random directions

$$\begin{aligned}
 S &= \frac{1}{4\pi} \int d\Omega \sin\theta \, d\theta \, (\cos^2\theta - 1/3) \\
 &= \frac{1}{2} \int_0^\pi d\theta \sin\theta (\cos^2\theta - 1/3) \\
 &= \frac{1}{2} \int_{-1}^1 d\eta (\eta^2 - 1/3) = \frac{1}{2} \left(\frac{1}{3}\eta^3 - \frac{1}{3}\eta \right) \Big|_{-1}^1 = 0
 \end{aligned}$$

perfect nematic order

$$\theta = \text{const}$$

$$S = 1 - \frac{1}{3} = \frac{2}{3}$$

b) $\langle \cos\theta \rangle$ is not an order parameter for the nematic because the nematic order is unchanged by $\theta \rightarrow \theta + \pi$ (symmetric rods) while $\langle \cos\theta \rangle$ changes sign.

c) Order parameters must be invariant under 90° rotations

4.1c) continued

$$S' = \langle \cos(4\theta) + C \rangle$$

to find C , consider random directions

$$S' = \frac{1}{4\pi} \int d\varphi \int_{\sin\theta} d\theta (\cos 4\theta + C)$$

$$= \frac{1}{2} \int_0^{\pi} d\theta \sin\theta (\cos 4\theta + C)$$

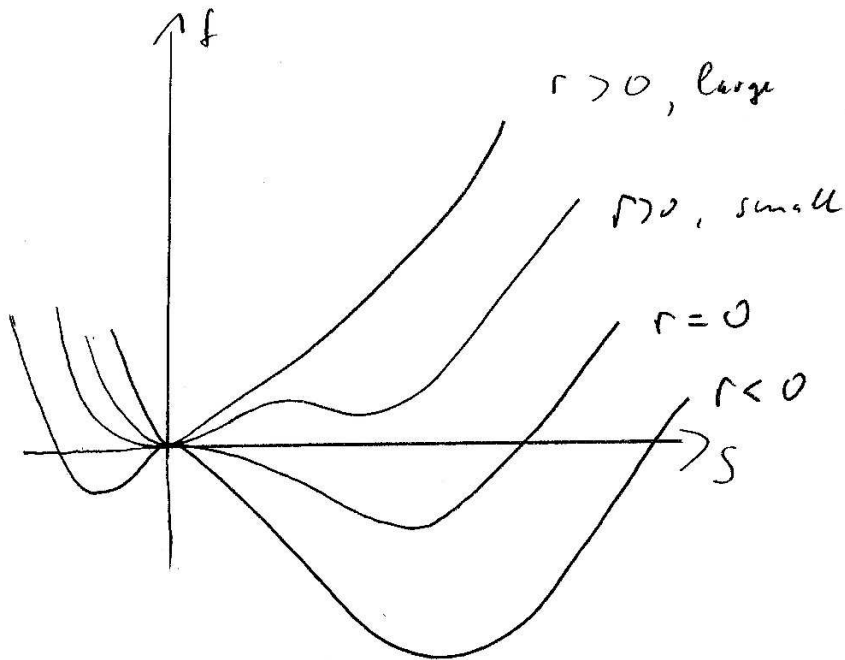
$$= \frac{1}{2} \left(-\frac{2}{15} + 2C \right) \quad \Rightarrow \quad C = \frac{1}{15}$$

$$S' = \langle \cos(4\theta) + \frac{1}{15} \rangle$$

C.f. for hemispherical case $S' = \langle \cos(2\theta) + \frac{1}{3} \rangle$

Problem 4.2.

a) set $u = w = 1$



b) $\frac{\partial f}{\partial s} = r s - 3 w s^2 + 4 u s^3 = s(r - 3 w s + 4 u s^2)$

$$s_1 = 0$$

$$s_{2,3} \neq s^2 - \frac{3w}{4u} s + \frac{\Gamma}{4u} = 0$$

$$s_{2,3} = \frac{3w}{8u} \pm \sqrt{\frac{9w^2}{64u^2} - \frac{\Gamma}{4u}}$$

c) Nematic state:

$$S = \frac{3}{8} \frac{w}{u} + \sqrt{\frac{9}{64} \frac{w^2}{u^2} - \frac{\Gamma}{4u}}$$

To find Γ_c :

$$f = 0$$

$$\frac{1}{2} \Gamma - wS + us^2 = 0 \quad \text{I}$$

$$\frac{\partial f}{\partial S} = 0$$

$$\Gamma - 3ws + 4us^2 = 0 \quad \text{II}$$

$$4\text{I} - \text{II} = \Gamma - ws = 0$$

$$\Gamma = ws$$

$$2\text{I} - \text{II} = ws - 2us^2 = 0$$

$$w = 2us$$

$$\boxed{S_c = \frac{w}{2u}, \quad \Gamma_c = \frac{w^2}{2u}}$$

d) isotropic phase locally stable for $\Gamma > 0$
nematic phase locally stable for

$$\frac{\Gamma}{4u} < \frac{9}{64} \frac{w^2}{u^2}$$

$$\Gamma^* = \frac{9}{16} \frac{w^2}{u}$$