Radioactive Decay

Radioactive decay is a random process during which atoms decay spontaneously and independently of each other. The probability that a certain atom decays during a small time interval $dt$ is given by

$$dP = \lambda \ dt$$

where the decay constant $\lambda$ is related to the half life $t_{1/2}$ by $\lambda = \ln(2)/t_{1/2}$. Consider an ensemble of $N_0$ radioactive atoms and simulate the stochastic process of the decay using pseudo random numbers. Calculate average, standard deviation and distribution function of the number of surviving atoms. Compare the simulation with your theoretical expectations.

a) Write a program which simulates the decay process of $N_0$ radioactive atoms with half life $t_{1/2}$ over a certain time $t_{\text{max}} \gg t_{1/2}$. Define a small time interval $\delta t$ with $\delta t \ll t_{1/2}$ (Why?), and use a pseudo random number generator to decide whether or not an individual atom decays during a certain slice. Repeat the entire simulation $R$ times with different random number seeds. Calculate the average number $\langle N(t) \rangle$ of surviving particles after each time slice and its standard deviation $\sigma(t)$.

b) Run the simulation for $N_0 = 1000$, $R = 1000$. Think about what values to choose for $t_{\text{max}}$ and $\delta t$. Plot the number of surviving atoms for the first 10 runs as functions of time together with the theoretically expected values. Plot the average $\langle N(t) \rangle$ and the standard deviation $\sigma(t)$. Plot and discuss the ratio $\sigma^2(t)/\langle N(t) \rangle$.

c) Calculate the distribution function of the number of surviving particles after a time $t_{\text{dis}}$. To do so, define a histogram having $N_{\text{bin}}$ bins of width $w_{\text{bin}}$. Find reasonable values for these two parameters. Plot the histogram.