Physics 6311: Statistical Mechanics - Homework 10

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due date: Tuesday, April 16, 2019

Problem 1: Liquid ^{3}Helium (6 points)

Liquid ^{3}He is approximately a Fermi gas (spin 1/2). The density is 0.081 g/cm^3.

a) Calculate the Fermi energy (at zero temperature). Also calculate the Fermi velocity (the velocity corresponding to the Fermi energy).

b) At roughly what temperatures do you expect the fermionic character of ^{3}He to be important?

Problem 2: Velocity distribution of the Fermi gas (10 points)

For an ideal Fermi gas at zero temperature, derive the probability density of the particle velocities and compare it to the Maxwell distribution of a classical ideal gas of the same total energy (per particle).

[Hint: You will need to find the correct temperature for the classical gas.]

Problem 3: Spin susceptibility of an ideal Fermi gas (12 points)

An ideal gas of N spin-1/2 fermions in a cube of of size L is under the influence of a weak magnetic field B. The field adds the term $\sigma \mu_B B$ to the single-particle energies where $\sigma = \pm 1$ for up and down spins, respectively. Neglect the effects of the field on the orbital motion of the fermions.

a) Find the Fermi momenta $k_{F\uparrow}$ and $k_{F\downarrow}$ for the up and down spins. (Because the field is weak, you can assume $\mu_B B \ll \epsilon_F$.)

b) Determine the magnetization $m = (\langle N_\uparrow \rangle - \langle N_\downarrow \rangle)/N$ at zero temperature. Here $\langle N_\uparrow \rangle$ and $\langle N_\downarrow \rangle$ are the numbers of spin-up and spin-down particles, respectively.

c) Determine the magnetic susceptibility (the so-called Pauli susceptibility) $\chi = (\partial m / \partial B)_T$.

Problem 4: Specific heat of Graphene (12 points)

Graphene is a two-dimensional form of carbon. Consider a square sheet of Graphene of linear size L. Two bands of single-particle electronic states are important at low temperatures, they have energies $\epsilon_+(\mathbf{k}) = \hbar v |\mathbf{k}|$ and $\epsilon_-(\mathbf{k}) = -\hbar v |\mathbf{k}|$. Here, v is a constant.

a) At zero temperature, all single-particle states with positive energies are empty while all states are with negative energies are occupied. Find the Fermi energy.

b) Calculate the chemical potential μ as function of T. [Hint: First show that in any Fermi gas, the probability of finding an occupies state at energy $\mu + \delta$ equals the probability of finding an empty state at energy $\mu - \delta$.]

c) Find the total energy and the specific heat as functions of temperature.