

# Physics 6311: Statistical Mechanics - Homework 12

---

due date: Tuesday, April 30, 2019

## Problem 1: Ising model with next-nearest neighbor interactions (40 points)

Consider an Ising model ( $S_i = \pm 1$ ) on a square lattice. There are interactions  $J$  and  $K$  between nearest neighbors and next-nearest neighbors as shown in the picture.

$$H = -J \sum_{\langle ij \rangle} S_i S_j - K \sum_{\{ij\}} S_i S_j - \mu_B B \sum_i S_i$$

where the first sum runs over all pairs of nearest neighbors and the second sum runs over all pairs of next-nearest neighbors.

- What is the ground state for large field  $\mu_B B \gg |J|, |K|$ ?
- Consider the case  $J > 0$  and  $K > 0$ . What is the ground state in the absence of a field ( $B = 0$ )? Find the ground state energy?.
- For  $J > 0$  and  $K > 0$ , determine the mean-field approximation for this Hamiltonian. Solve the mean-field Hamiltonian and derive the mean-field equation.
- Solve this mean-field equation for  $B = 0$  and find the critical temperature.
- Calculate the resulting critical exponents  $\beta, \gamma$ , and  $\delta$  for the magnetization, susceptibility, and the critical isotherm, respectively.
- Determine the ground state for the case  $J < 0, K > 0$  and zero field. Find the ground state energy.
- Consider the phase transition that occurs as a function of  $J$  at fixed  $T = 0, B = 0, K > 0$ . Is it first order or continuous.
- Discuss qualitatively what happens for  $J > 0, K < 0$  or  $J < 0, K < 0$ . (The field  $B = 0$ .)
- Sketch the ground-state phase diagram as function of  $J$  and  $K$ .

