Problem 1: Carnot process for a paramagnetic substance (16 points)

Consider a paramagnetic substance whose equation of state is $M = DH/T$ where $T$ is the temperature, $M$ is the magnetization, $H$ is magnetic field, and $D$ is a material specific constant. The internal energy is $U = CT$ where the specific heat $C$ is a constant.

a) Sketch a typical Carnot cycle in the M-H plane.
b) Compute the total absorbed heat and the total work during one cycle.
c) Calculate the efficiency.

Problem 2: Entropy of the ideal gas (8 points)

The equation of state of an ideal gas is $pV = Nk_B T$ with $p$ being pressure, $V$ volume, $N$ the number of particles, $k_B$ the Boltzmann constant, and $T$ the temperature. The internal energy is $U = (3/2)Nk_B T$. Calculate the entropy of the ideal gas as a function of $T$ and $V$. What happens for $T \to 0$?

Problem 3: Maxwell relations (8 points)

For a dielectric material characterized by polarization $P$, electric field $E$, and temperature $T$ (PET system), derive the so-called Maxwell relations between various derivatives of thermodynamic quantities. The Maxwell relations follow from the fact that the differentials of the thermodynamic potentials $U$, $H$, $A$ and $G$ are exact, if they are expressed in terms of their natural variables.

Problem 4: Rubber elasticity (8 points)

The equation of state of a rubber band can be modeled by the so-called Guth-James equation

$$F = aT \left[ \frac{L}{L_0} - \frac{L_2^2}{L_1^2} \right].$$

Here $F$ is the tension force, $L$ is the length of the rubber band (with $L_0$ being the unstretched length). $T$ is temperature, and $a$ is a positive constant.

a) Derive an expression for the work done in stretching the rubber band isothermally from length $L_1$ to length $L_2$.
b) When the rubber band is heated at fixed tension, will its length increase or decrease with temperature? Base your answer on the equation of state.