Physics 6311: Statistical Mechanics - Homework 2

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due date: Tuesday, Feb 5, 2019

Problem 1: Carnot process for a paramagnetic substance (16 points)

Consider a paramagnetic substance whose equation of state is $M = DH/T$ where T is the temperature, M is the magnetization, H is magnetic field, and D is a material specific constant. The internal energy is $U = CT$ where the specific heat C is a constant.

a) Sketch a typical Carnot cycle in the M-H plane.
b) Compute the total absorbed heat and the total work during one cycle.
c) Calculate the efficiency.

Problem 2: Entropy of the ideal gas (8 points)

The equation of state of an ideal gas is $pV = Nk_BT$ with p being pressure, V volume, N the number of particles, k_B the Boltzmann constant, and T the temperature. The internal energy is $U = (3/2)Nk_BT$. Calculate the entropy of the ideal gas as a function of T and V. What happens for $T \to 0$?

Problem 3: Maxwell relations (8 points)

For a dielectric material characterized by polarization P, electric field E, and temperature T (PET system), derive the so-called Maxwell relations between various derivatives of thermodynamic quantities. The Maxwell relations follow from the fact that the differentials of the thermodynamic potentials U, H, A and G are exact, if they are expressed in terms of their natural variables.

Problem 4: Rubber elasticity (8 points)

The equation of state of a rubber band can be modeled by the so-called Guth-James equation

$$F = aT\left[\frac{L}{L_0} - \frac{L_0^2}{L^2}\right] .$$

Here F is the tension force, L is the length of the rubber band (with L_0 being the unstretched length). T is temperature, and a is a positive constant.

a) Derive an expression for the work done in stretching the rubber band isothermally from length L_1 to length L_2.
b) When the rubber band is heated at fixed tension, will its length increase or decrease with temperature? Base your answer on the equation of state.