# Physics 6311: Statistical Mechanics - Homework 3

due date: Tuesday, Feb 12, 2019

**Problem 1: Power-law distribution** (8 point)

The probability density of a random variable $J$ has the power-law form

$$P_J(j) = \begin{cases} A\, j^y & (0 \le j \le 1) \\ 0 & \text{otherwise} \end{cases} .$$

where $A$ and $y$ are constants.

a) Find the normalization constant $A$ as a function of the exponent $y$. For which $y$ is $P_J$ a valid probability density?

b) Calculate the average of $J$ as well as the geometric mean. How do they compare for $y \to -1$?

**Problem 2: Transforming random variables** (14 points)

The random variables $X$ and $Y$ are independent and have identical box distributions

$$P_X(x) = \begin{cases} 1/2 & (|x| < 1) \\ 0 & (|x| > 1) \end{cases} , \quad P_Y(y) = \begin{cases} 1/2 & (|y| < 1) \\ 0 & (|y| > 1) \end{cases}$$

a) Find the characteristic functions for $X$ and $Y$.

b) Find the characteristic function for the random variable $Z = X + Y$.

c) Derive the probability density $P_Z(z)$ of the random variable $Z$

d) Compute the moments $\langle z \rangle$, $\langle z^2 \rangle$, $\langle z^3 \rangle$, and $\langle z^4 \rangle$

**Problem 3: Distribution of the minimum of several random numbers** (8 points)

Consider $N$ random variables $X_i$ $(i = 1 \ldots N)$ chosen independently from a uniform (box) probability density on the interval $[-1, 1]$. Let $m$ be the minimum of the $N$ values, $m = \min(x_1, \ldots, x_N)$.

a) Calculate the probability density $P_N(m)$ of $m$. Hint: It may be easier to think about the distribution $F_N(m)$ of $m$ first.

b) Calculate the mean and variance of $m$ as functions of $N$. How do they behave for $N \to \infty$?

**Problem 4: Diode** (10 points)

The current $I$ across a diode is related to the applied voltage $V$ via

$$I = I_0[e^{eV/(k_B T)} - 1] .$$

The diode is subject to a random voltage $V$ which is Gaussian distributed with zero mean and variance $\sigma^2$. Calculate the probability density $P(I)$ of the current. Find the most probable current and the average current.