

Physics 6311: Statistical Mechanics - Homework 3

due date: September 14, 2021

Problem 1: Probability of a density fluctuation (10 points)

Consider two identical boxes, A and B.

- 10 particles are distributed over the two boxes at random. Calculate the probabilities $P(4)$ and $P(5)$ for finding exactly $N_A = 4$ and $N_A = 5$ particles in the box A, respectively. Calculate $P(4)/P(5)$.
- Repeat the calculations for 10^{23} particles. Compare $N_A = 0.4 * 10^{23}$ and $N_A = 0.5 * 10^{23}$. (For large n the factorial can be approximated by Stirling's formula $n! = \sqrt{2\pi n} n^n e^{-n}$)

Problem 2: Transformation of random variables (10 points)

The random variables X and Y are independent and Gaussian distributed with averages $\langle x \rangle = \langle y \rangle = 0$ and standard deviations $\sigma_x = \sigma_y = 1$.

- Calculate the characteristic functions of X and Y .
- Find the characteristic function for the random variable $Z = X^2 + Y^2$.
- Derive the probability density $P_Z(z)$ of the random variable Z
- Compute the moments $\langle z \rangle$, $\langle z^2 \rangle$, and $\langle z^3 \rangle$. Find the first three cumulants.

Problem 3: Power-law distribution (10 point)

The probability density of a random variable J has the power-law form

$$P_J(j) = \begin{cases} A(1-j)^y & (0 \leq j \leq 1) \\ 0 & \text{otherwise} \end{cases} .$$

where A and y are constants.

- Find the normalization constant A as a function of the exponent y . For which y is P_J a valid probability density?
- Calculate the average of J as well as the geometric mean. How do they compare for $y \rightarrow -1$?

Problem 4: Diode (10 points)

The current I across a diode is related to the applied voltage V via

$$I = I_0 [e^{eV/(k_B T)} - 1] .$$

The diode is subject to a random voltage V which is Gaussian distributed with zero mean and variance σ^2 . Calculate the probability density $P(I)$ of the current. Find the most probable current and the average current.