Physics 6311: Statistical Mechanics - Homework 4

due date: Tuesday, Feb 19

Problem 1: Shannon entropy of independent random variables (8 points)

Consider the joint probability distribution $P_{X_1 \ldots X_N}$ of N random variables $X_1, \ldots, X_N$. Show that if the X_i are statistically independent then the Shannon entropy S_s of their joint distribution is the sum of the Shannon entropies of the reduced distributions P_{X_i} of the individual variables X_i.

Problem 2: Shannon entropy of N spin-2 atoms (5 points)

Consider a lattice with $N \gg 1$ spin-2 atoms. Each atom can be in one of the five spin states $S_z = -2, -1, 0, +1, +2$ with equal probability. Calculate the Shannon entropy of this system.

Problem 3: Maxima of entropy (12 points)

Consider the entropy of a discrete probability distribution given in terms of the probabilities p_i ($i = 1 \ldots N$). Determine which p_i lead to the maximum entropy under the following constraints (Hint: Use Lagrange multipliers to enforce the constraints.):

a) Normalization $\sum_i p_i = 1$

b) Normalization $\sum_i p_i = 1$ and fixed average $\langle a \rangle = \sum_i p_i a_i$ of a quantity A with values a_i.

c) Normalization, fixed average $\langle a \rangle$ and fixed variance σ_A^2.

Problem 4: Atoms on a lattice (15 points)

Consider a lattice having N regular lattice sites as well as N interstitial lattice sites. The lattice is occupied by N identical atoms. An atom on a regular site has energy 0 while an atom on an interstitial site has energy ϵ. Use the microcanonical ensemble to calculate the internal energy and the specific heat as functions of temperature.