Problem 1: Generalized equipartition theorem (10 points)

Consider a classical Hamiltonian of the form

\[ H = \sum_{i=1}^{3N} \frac{1}{2} A_i |q_i|^n + \sum_{i=1}^{3N} \frac{1}{2} B_i p_i^2 \]

where \( n > 0 \) is an exponent that characterizes the potential energy and \( A_i \) and \( B_i \) are positive constants. Using the canonical ensemble, calculate the internal energy and the specific heat at constant volume as functions of temperature.

Problem 2: Ideal gas with movable piston (15 points)

A classical ideal gas of \( N \) particles is in a cylindrical vessel of cross section \( A \). The top of the vessel is closed by a movable piston of mass \( M \).

a) Calculate the partition function for the system consisting of gas + piston in the canonical ensemble.

b) Determine the equation of state, the average volume and the heat capacity.

c) Discuss which heat capacity you are actually calculating.

Problem 3: Broadening of spectral lines (15 points)

The atoms of a star emit light. The emission frequency of a particular element is \( \nu_0 \) if the atom is a rest. Due to the thermal motion (temperature \( T \)) the observed frequency is shifted (Doppler effect) to

\[ \nu = \nu_0 \left(1 - \frac{v}{c} \cos \theta \right) \]

where \( v \) is the velocity of the atom and \( \theta \) is the angle between the directions of motion and observation. Calculate the resulting intensity distribution \( \rho(\nu) \). What is the width of the spectral line? (Assume the atoms to be noninteracting and to move non-relativistically!)