Physics 6311: Statistical Mechanics - Homework 8

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due date: Tuesday, April 2, 2019

Problem 1: Identical particles in two-level system (10 points)

A quantum mechanical system has two single-particle states $|a\rangle$ and $|b\rangle$ with energies $\epsilon_a = -\epsilon_b = \epsilon$.

a) The system is occupied by two identical particles. Write down all possible states, the corresponding energies and the canonical probabilities for these states for bosons (S=0) and for fermions (S=1/2, but both particles being in the $\uparrow$ state). Using the canonical ensemble calculate the Helmholtz free energy, the entropy, the internal energy and the specific heat as functions of temperature.

b) Consider an additional term in the Hamiltonian, viz, an interaction between the particles of the form $U\, n_a\, n_b$. where U is the interaction energy and n_a and n_b are the particle numbers of the two single-particle states. How do the canonical probabilities for the two-boson states from a) change as a result of U? Discuss the limits $U \to \infty$ and $U \to -\infty$.

Problem 2: Quantum corrections to classical ideal gas (10 points)

Calculate the lowest order quantum corrections to the energy of the classical ideal gas as function of particle number and temperature both for fermions and for bosons.
(Hint: Start from the Bose and Fermi occupation numbers. In the classical (Boltzmann) limit the average occupation numbers are small compared to 1. Expand the occupation numbers about this limit. Don't forget the evaluate μ.)

Problem 3: Bose-Einstein temperature of Helium (5 points)

The density of liquid ^{4}Helium is 0.145g/cm^3. Calculate the Bose-Einstein condensation temperature of an ideal Bose gas of the same number density and compare it with the superfluid temperature of 2.2K.

Problem 4: Generalized Bose gas (15 points)

Consider a gas of noninteracting identical bosons of spin S in d dimensions. The single-particle energy-momentum relation is given by $\epsilon(\mathbf{p}) = A|\mathbf{p}|^z$ with positive prefactor A and exponent z.

a) Compute the density of states $g(\epsilon)$.

b) Calculate the maximum possible particle number in excited single-particle states as a function of temperature. For which values of d and z does the system show Bose-Einstein condensation?

c) If there is Bose-Einstein condensation, evaluate the critical temperature T_c

d) Find the specific heat for temperatures $T \leq T_c$.

e) Find the pressure for temperatures $T \leq T_c$.