Physics 6311: Statistical Mechanics - Homework 9

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due date: Tuesday, April 9, 2019

Problem 1: Radiation of Betelgeuse (8 points)

The luminosity (total amount of energy emitted per time) of the star Betelgeuse is about 10^4 times that of the sun. The energy density $u(\epsilon)$ of its radiation has its maximum at a photon energy $\epsilon \approx 0.8$ eV.

a) Find the surface temperature of Betelgeuse, assuming it emits blackbody radiation.

b) Estimate the radius of Betelgeuse.

c) Why is Betelgeuse called a red giant?

Problem 2: Phonons in liquid ^{4}Helium (8 points)

The longitudinal phonons in ^{4}He at low temperatures have a velocity of $c = 238.3$ m/s. Transversal phonons do not exist in liquids. The density is 0.145 g/cm^3.

a) Calculate the Debye temperature (within the Debye model).

b) Calculate the heat capacity and compare to the experimental value of $c_V = 0.0204$ (T/K)^3 J/gK.

Problem 3: Thermodynamics of Magnons (12 points)

Spin waves or magnons are elementary excitations of Bose type in ferromagnetic materials. Their dispersion relation is $\omega = D\ k^2$ for small frequencies $\omega \ll \omega_{max}$. Calculate the contribution of the magnons to the specific heat at low temperatures $k_B T \ll \hbar\omega_{max}$. (Hint: There is no conservation law for the magnon number, the rest mass is zero.)

Problem 4: Phonons in a 1D chain (12 points)

Consider a one-dimensional chain of atoms (model for a linear molecule). The vibrational part of the Hamiltonian is

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \frac{A}{2} \sum_{i=1}^{N} (x_i - x_{i+1})^2 \ .$$

where x_i is the displacement of atom i and m is the mass of one of the atoms. Assume periodic boundary conditions.

a) Determine the normal modes by diagonalizing H (Hint: Use the Fourier transformation).

b) Calculate energy and specific heat as functions of temperature for low temperatures.