## Ising model with next-nearest neighbor interactions

a) $\mu_B B \gg |J|, |K|$

spins align with field

b) all spins align with each other, either all up or all down

$$E_0 = -2NJ - 2NK$$

c) $$H_{MF} = -\sum_i (4mJ + 4mK + \mu_B B) S_i$$

$$m = \langle S_i \rangle$$

$$m = \tanh\left[\beta(4mJ + 4mK + \mu_B B)\right]$$

d) $T_c$ given by $\beta(4J + 4K) = 1$

$$k_B T_c = 4J + 4K$$

e) $$\underbrace{\text{Artanh}(m)}_{\text{Expand}} = \beta\mu_B B + 4m\beta(J+K)$$

$$m\frac{T-T_c}{T_c} + \frac{1}{3}m^3 = \frac{\mu_B B}{k_B T_c}$$

as in simple Ising case

$$\beta = \frac{1}{2}, \quad \delta = 3, \quad \gamma = 1$$

f) ground state is antiferromagnet

↑↓↑ $\quad E_0 = 2NJ - 2NK$

↓↑↓ $\quad = -2N|J| - 2NK$

g) ground state jumps from ferromagnet
↳ antiferromagnet as $J$ moves through 0
⇒ 1st order

h) $J > 0, K < 0$ or $J < 0, K \leq 0$
not all bonds can be fulfilled
⇒ frustration ⇒ complicated behavior