model with next-nearest neighbor interactions

a) $\mu_B B \gg |J|, |K|$
spins align with field

b) all spins align with each other, either all up or all down
$E_0 = -2NJ - 2NK$

c) $H_{AF} = -\sum \left( 4mJ + 4mK + p_0 \hat{B} \right) S_i$
$m = \langle S_i \rangle$

$m = \tanh \left[ \beta \left( 4mJ + 4mK + p_0 \hat{B} \right) \right]$

d) $T_c$ given by $\beta \left( 4J + 4K \right) = 1$

$k_B T_c = 4J + 4K

e) \frac{\text{Arctanh} \left( m \right)}{m} = \beta p_0 \hat{B} + 4m \beta \left( J + K \right)$

Expand

$m \frac{T_c}{T_c} + \frac{1}{3} m^3 = \frac{\mu_B B}{k_B T_c}$

as in simple Ising case

$\beta = \frac{1}{T_c}$, $S = 3$, $\tau = 1$
f) ground state is antiferromagnet

\[ \sum_{i} E_{i} = 2NJ - 2NK \]

\[ \sum_{ij} = -2NJij - 2NK \]

9) ground state jumps from ferromagnet to anti-ferromagnet as J moves through 0

\( \Rightarrow \) 1st order

b) \( J > 0, K < 0 \) or \( J < 0, K > 0 \)
not all bonds can be fulfilled

\( \Rightarrow \) frustration \( \Rightarrow \) complicated behavior