Problem 1: Carnot process for paramagnetic substance

Isotherms are straight lines, form of adiabats follows from below: \( CD \ln(T/T_0) = (M^2 - M_0^2)/2 \).

Substitute for \( T \): \( H/H_0 = (M/M_0) \exp[(M^2 - M_0^2)/(2CD)] \).

1—2 isothermal at \( T_1 \): \( dU = 0 \)

\[ \Delta Q_{12} = -\Delta W = -\int_{M_1}^{M_2} H dM = \int_{M_1}^{M_2} \frac{M}{CD} dM = -\frac{T_1}{2CD}(M_2^2 - M_1^2) \]

3—4 isothermal at \( T_2 \), analogously: \( \Delta Q_{34} = -\frac{T_2}{2CD}(M_4^2 - M_3^2) \)

2—3 adiabatic, \( \delta Q = 0 = dU - HdM = CdT - HdM \)

Use equation of state to substitute \( H \):

\( CDDT/T = MDM \)

\( CD \ln(T_2/T_1) = (M_2^2 - M_1^2)/2 \)

4—1 analogously \( CD \ln(T_1/T_2) = (M_1^2 - M_2^2)/2 \)

Combining the two adiabatic parts gives \( M_3^2 - M_4^2 = -(M_1^2 - M_2^2) \).

\[ \eta = 1 + \frac{\Delta Q_{34}}{\Delta Q_{12}} = 1 + \frac{T_2(M_4^2 - M_2^2)}{T_1(M_1^2 - M_2^2)} = 1 - \frac{T_2}{T_1} \]
Problem 2: Entropy of the ideal gas

\[ pV = Nk_B T, \quad U = (3/2)Nk_B T \]

\[ dU = TdS - pdV \]

\[ dS = dU/T + pdV/T = (3/2)Nk_B dT/T + Nk_B dV/V \]

\[ S = (3/2)Nk_B \log(T/T_0) + Nk_B \log(V/V_0) + S_0 \]

\( S \) diverges for \( T \to 0 \): violation of the third law. Ideal gas ill defined for \( T \to 0 \).

Problem 3: Maxwell relations

\[ dU = T \, dS + E \, dP: \quad (\partial T/\partial P)_S = (\partial E/\partial S)_P \]

\[ dH = T \, dS - P \, dE: \quad (\partial T/\partial E)_S = -(\partial P/\partial S)_E \]

\[ dA = -S \, dT + E \, dP: \quad (\partial S/\partial P)_T = -(\partial E/\partial T)_P \]

\[ dG = -S \, dT - P \, dE: \quad (\partial S/\partial E)_T = (\partial P/\partial T)_E \]
Problem 4: Rubber elasticity

\[ dW = F \, dL \]
\[ = \alpha \tau \left[ \frac{L}{L_0} - \frac{L_0^2}{L} \right] \, dL \]

\[ \Delta W = \alpha \tau \left[ \frac{1}{2} \frac{L_0^2}{L_0} + \frac{L_0^2}{L} \right] \frac{L_0^2}{L} \]

\[ \Delta W = \alpha \tau \left[ \frac{1}{2} \frac{L_0^2}{L_0} - \frac{1}{2} \frac{L_0^2}{L_0} + \frac{L_0^2}{L_0} - \frac{L_0^2}{L_1} \right] \]

5) \quad L_0 = L_1 = 2.0 \text{ cm} \\
L_2 = 50 \text{ cm} \\
\tau = 293 \text{ K} \\
\alpha = 1.33 \times 10^{-2} \text{ N/K} \\

\[ \Delta W = \alpha \tau \left[ 0.405 \text{ m} \right] \\
= 1.578 \text{ N/m} \]

\( F = \text{ const.} \) implies that

\[ [\ ] \text{ decrease as } \tau \text{ increase} \]

\[ \frac{d}{dL} [\ ] = \frac{1}{L_0} + 2 \frac{L_0^2}{L_0^2} > 0 \]

\( \Rightarrow \) rubber band shrinks as \( \tau \) increase