Strong-disorder ferromagnetic quantum phase transitions

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Collective modes at a disordered quantum phase transition

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Outline

- Collective modes: Goldstone and amplitude (Higgs)
- Superfluid-Mott glass quantum phase transition
- Fate of the collective modes at the superfluid-Mott glass transition
- Conclusions
Broken symmetries and collective modes

- collective excitation in systems with **broken continuous symmetry**, e.g.,
  - planar magnet breaks $O(2)$ rotation symmetry
  - superfluid wave function breaks $U(1)$ symmetry

- **Amplitude (Higgs) mode**: corresponds to fluctuations of order parameter **amplitude**

- **Goldstone mode**: corresponds to fluctuations of order parameter **phase**

- **Amplitude mode** is condensed matter analogue of famous **Higgs boson**

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effective potential for order parameter in symmetry-broken phase
What is the fate of the Goldstone and amplitude modes near a disordered quantum phase transition?
• Collective modes: Goldstone and amplitude (Higgs)

• **Superfluid-Mott glass quantum phase transition**

• Fate of the collective modes at the superfluid-Mott glass transition

• Conclusions
Disordered interacting bosons

Ultracold atoms in optical potentials:
- disorder: speckle laser field
- interactions: tuned by Feshbach resonance and/or density

F. Jendrzejewski et al., Nature Physics 8, 398 (2012)

Disordered superconducting films:
- energy gap in insulating as well as superconducting phase
- preformed Cooper pairs ⇒ superconducting transition is bosonic

Disordered interacting bosons

Bosonic quasiparticles in doped quantum magnets:

- bromine-doped dichloro-tetrakis-thiourea-nickel (DTN)
- coupled antiferromagnetic chains of $S = 1$ Ni$^{2+}$ ions
- $S = 1$ spin states can be mapped onto bosonic states with $n = m_s + 1$

Yu et al., Nature 489, 379 (2012)
Bose-Hubbard model

Bose-Hubbard Hamiltonian in two dimensions:

\[
H = \frac{U}{2} \sum_i (\hat{n}_i - \bar{n}_i)^2 - \sum_{\langle i,j \rangle} J_{ij} (a_i^\dagger a_j + h.c.)
\]

- superfluid ground state if Josephson couplings \( J_{ij} \) dominate
- insulating ground state if charging energy \( U \) dominates
- chemical potential \( \mu_i = U \bar{n}_i \)

Particle-hole symmetry:

- large integer filling \( \bar{n}_i = k \) with integer \( k \gg 1 \)
  \( \Rightarrow \) Hamiltonian invariant under \( (\hat{n}_i - \bar{n}_i) \to -(\hat{n}_i - \bar{n}_i) \)
Stability of clean quantum critical point against dilution

**Site dilution:**

- randomly remove a fraction \( p \) of lattice sites
- superfluid phase possible for \( 0 \leq p \leq p_c \) (percolation threshold)

**Harris criterion:**

- for dilution \( p = 0 \), quantum critical point is in 3D XY universality class
- correlation length critical exponent \( \nu \approx 0.6717 \)
- clean \( \nu \) violates Harris criterion \( d\nu > 2 \) with \( d = 2 \)

\[ \Rightarrow \text{clean critical behavior unstable against disorder (dilution)} \]

Critical behavior of superfluid-Mott glass transition must be in new universality class.
Monte Carlo simulations

- large-scale Monte Carlo simulations in 2d and 3d
- **conventional** critical behavior with universal exponents for dilutions $0 < p < p_c$

### (2+1)D exponents

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<th>disordered</th>
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### (3+1)D exponents

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</table>
• Collective modes: Goldstone and amplitude (Higgs)
• Superfluid-Mott glass quantum phase transition
• **Fate of the collective modes at the superfluid-Mott glass transition**
• Conclusions
Amplitude mode: scalar susceptibility

- Parameterize order parameter fluctuations into \( \phi = \phi_0 (1 + \rho) \hat{n} \)

- Amplitude mode is associated with scalar susceptibility

\[
\chi_{\rho\rho}(\vec{x}, t) = i\Theta(t) \langle [\rho(\vec{x}, t), \rho(0, 0)] \rangle
\]

- Monte-Carlo simulations compute imaginary time correlation function

\[
\chi_{\rho\rho}(\vec{x}, \tau) = \langle \rho(\vec{x}, \tau) \rho(0, 0) \rangle
\]

- \textbf{Wick rotation} required: analytical continuation from imaginary to real times/frequencies \( \Rightarrow \text{maximum entropy method} \)
Amplitude mode in clean undiluted system

Scaling form of the scalar susceptibility: [Podolsky + Sachdev, PRB 86, 054508 (2012)]

\[ \chi_{\rho\rho}(\omega) = |r|^{3\nu - 2} X(\omega |r|^{-\nu}) \]

- sharp Higgs peak in spectral function
- Higgs energy (mass) \( \omega_H \) scales as expected with distance from criticality \( r \)
Amplitude mode in disordered system

- spectral function shows broad peak near $\omega = 1$
- peak is noncritical: does not move as quantum critical point is approached
- amplitude fluctuations **not soft at criticality**
- **violates** expected scaling form $\chi_{\rho\rho}(\omega) = |r|^{(d+z)\nu-2} X(\omega|r|^{-z\nu})$
What is the reason for the absence of a sharp amplitude mode at the superfluid-Mott glass transition?
Quantum mean-field theory

\[ H = \frac{U}{2} \sum_i \epsilon_i (\hat{n}_i - \bar{n}_i)^2 - J \sum_{\langle i,j \rangle} \epsilon_i \epsilon_j (a_i^\dagger a_j + \text{h.c.}) \]

- truncate Hilbert space: keep only states \(|\bar{n} - 1\rangle, |\bar{n}\rangle, \text{ and } |\bar{n} + 1\rangle\) on each site

**Variational wave function:**

\[ |\Psi_{MF}\rangle = \prod_i |g_i\rangle = \prod_i \left[ \cos \left( \frac{\theta_i}{2} \right) |\bar{n}\rangle_i + \sin \left( \frac{\theta_i}{2} \right) \frac{1}{\sqrt{2}} \left( e^{i\phi_i} |\bar{n} + 1\rangle_i + e^{-i\phi_i} |\bar{n} - 1\rangle_i \right) \right] \]

- locally interpolates between **Mott insulator**, \(\theta = 0\), and **superfluid limit**, \(\theta = \pi/2\)

**Mean-field energy:**

\[ E_0 = \langle \Psi_{MF} | H | \Psi_{MF} \rangle = \frac{U}{2} \sum_i \epsilon_i \sin^2 \left( \frac{\theta_i}{2} \right) - J \sum_{\langle i,j \rangle} \epsilon_i \epsilon_j \sin(\theta_i) \sin(\theta_j) \cos(\phi_i - \phi_j) \]

- solved by **minimizing** \(E_0\) w.r.t. \(\theta_i \Rightarrow \) coupled nonlinear equations
• **local order parameter:** $m_i = \langle a_i \rangle = \sin(\theta_i)e^{i\phi_i}$  
  
  (dilution $p = 1/3$)
Mean-field theory: excitations

- **define local excitations** (orthogonal to $|g_i\rangle$, OP phase fixed at 0)

\[
|g_i\rangle = \cos \left( \frac{\theta_i}{2} \right) |\bar{n}\rangle_i + \sin \left( \frac{\theta_i}{2} \right) \frac{1}{\sqrt{2}} (|\bar{n} + 1\rangle_i + |\bar{n} - 1\rangle_i)
\]

\[
|\theta_i\rangle = \sin \left( \frac{\theta_i}{2} \right) |\bar{n}\rangle_i - \cos \left( \frac{\theta_i}{2} \right) \frac{1}{\sqrt{2}} (|\bar{n} + 1\rangle_i + |\bar{n} - 1\rangle_i)
\]

\[
|\phi_i\rangle = \frac{1}{\sqrt{2}} (|\bar{n} + 1\rangle_i - |\bar{n} - 1\rangle_i)
\]

- **expand H to quadratic order in excitations**: $H = E_0 + H_\theta + H_\phi$

\[
H_\theta = \sum_i \left[ \frac{U}{2} + 2J \sum_{j'} \sin(\theta_i) \sin(\theta_j) \right] \epsilon_i b_{\theta_i}^\dagger b_{\theta_i}
\]

\[
- J \sum_{\langle ij \rangle} \cos(\theta_i) \cos(\theta_j) \epsilon_i \epsilon_j (b_{\theta_i}^\dagger + b_{\theta_i})(b_{\theta_j}^\dagger + b_{\theta_j})
\]

$H_\phi$ has similar structure but different coefficients.
Clean system: results

- Mean-field quantum phase transition at $U = 16J$

- All excitations are spatially extended (plane waves)

**Mott insulator**

- All excitations are gapped

**Superfluid**

- Goldstone mode is gapless

- Amplitude (Higgs) modes is gapped, gap vanishes at QCP
Diluted lattice: Goldstone mode

- Goldstone mode becomes massless in superfluid phase, as required by Goldstone’s theorem
- Wave function of lowest excitation for $U = 8$ to 15
- Localized in insulator, delocalizes in superfluid phase
Goldstone mode: localization properties

- inverse participation ratio:
  \[ P^{-1} = N \sum_i |\psi_i|^4 \]

  \( P \to 1 \) for delocalized states
  \( P \to 0 \) for localized states

- wave function at \( U = 8 \) as function of excitation energy

- delocalized at \( \omega = 0 \), localized for higher energies
Amplitude (Higgs) mode

- amplitude mode strongly localized for all $U$ and all excitation energies

- wave function of lowest excitation for $U = 8$ to 15
Longitudinal and transverse susceptibilities \((q = 0)\)

diluted, \(p = 1/3\)

clean
Conclusions

- disordered interacting bosons undergo quantum phase transition between [superfluid state and insulating Mott glass state]

- *conventional* critical behavior with universal critical exponents


- collective modes in superfluid phase show *striking localization behavior*

- Goldstone mode is delocalized at $\omega = 0$ but localizes with increasing energy

- amplitude (Higgs) mode is strongly localized for all energies

- broad incoherent scalar response at $q = 0$, violates naive scaling

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Exotic collective mode dynamics even if critical behavior is conventional

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T.V., Jack Crewse, Martin Puschmann, Daniel Arovas, and Yury Kiselev, PRB 94, 134501 (2016)
**Analytic continuation - maximum entropy method**

- **Matsubara susceptibility** $\chi_{\rho\rho}(i\omega_m)$ vs. **spectral function** $A(\omega) = \chi''_{\rho\rho}(\omega)/\pi$

\[
\chi_{\rho\rho}(i\omega_m) = \int_0^\infty d\omega A(\omega) \frac{2\omega}{\omega_m^2 + \omega^2}
\]

**Maximum entropy method:**

- inversion is ill-posed problem, highly sensitive to noise
- fit $A(\omega)$ to $\chi_{\rho\rho}(i\omega_m)$ MC data by minimizing $Q = \frac{1}{2}\sigma^2 - \alpha S$
- parameter $\alpha$ balances between fit error $\sigma^2$ and entropy $S$ of $A(\omega)$, i.e., between fitting information and noise
- best $\alpha$ value chosen by L-curve method [see Bergeron et al., PRE 94, 023303 (2016)]