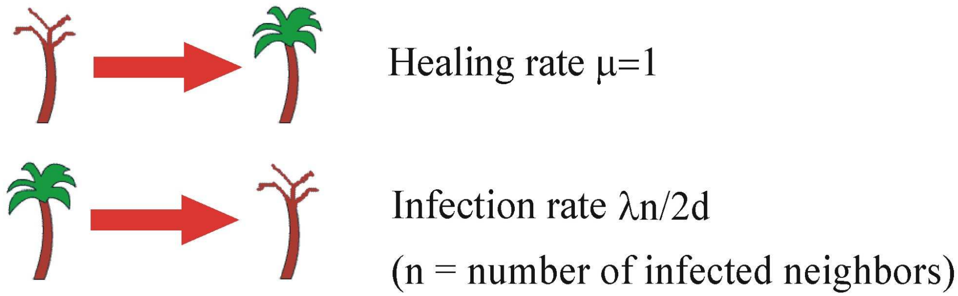
Rare region effects at a non-equilibrium phase transition with linear defects

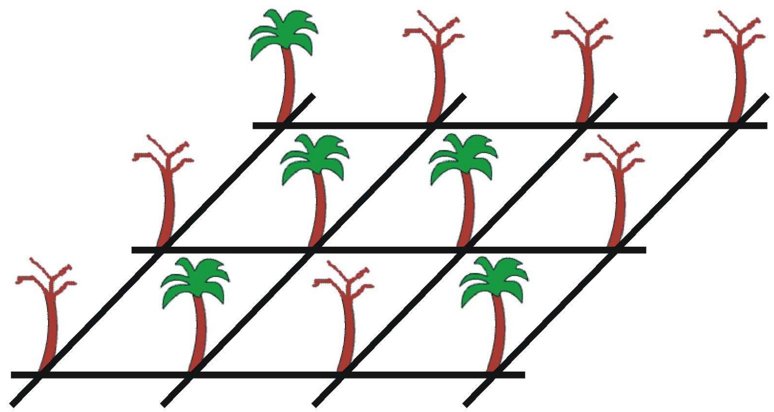
Mark Dickison and Thomas Vojta
Department of Physics, University of Missouri-Rolla

- Contact process and directed percolation
- Disorder and rare regions
- Smearing of the phase transition

Contact process

- prototypical nonequilibrium process
- model for the spreading of a disease





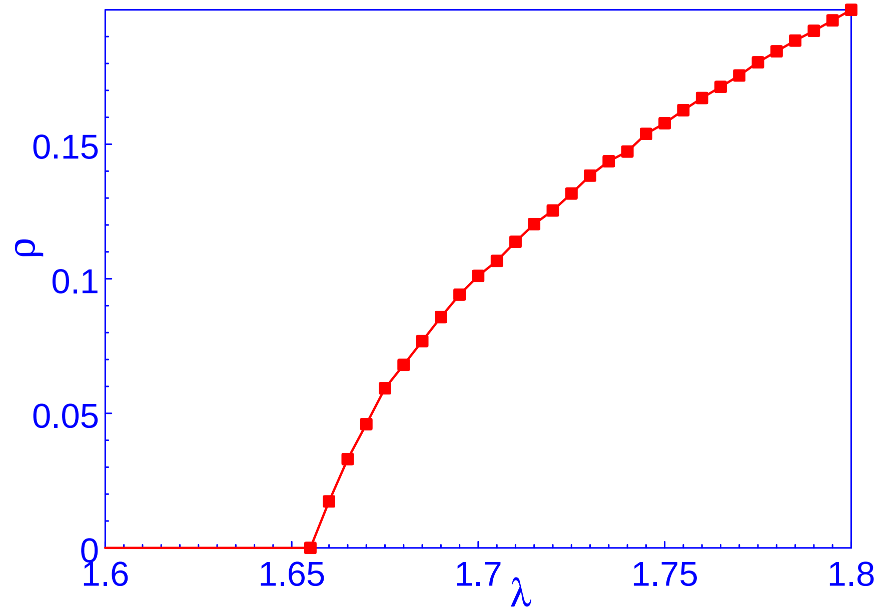
- defined on d-dimensional hypercubic lattice
- each site can be either healthy (inactive) or sick (active)
- sick sites heal spontaneously with rate μ
- healthy sites get infected with a rate $\lambda n/2d$ (where n is the number of sick neighbors)

Nonequilibrium phase transition in the clean contact process

- small infection rate λ:
  infection dies out, **absorbing state** with no active sites is the only steady state

- large infection rate λ:
  active steady state with nonzero fraction of sick (active) sites

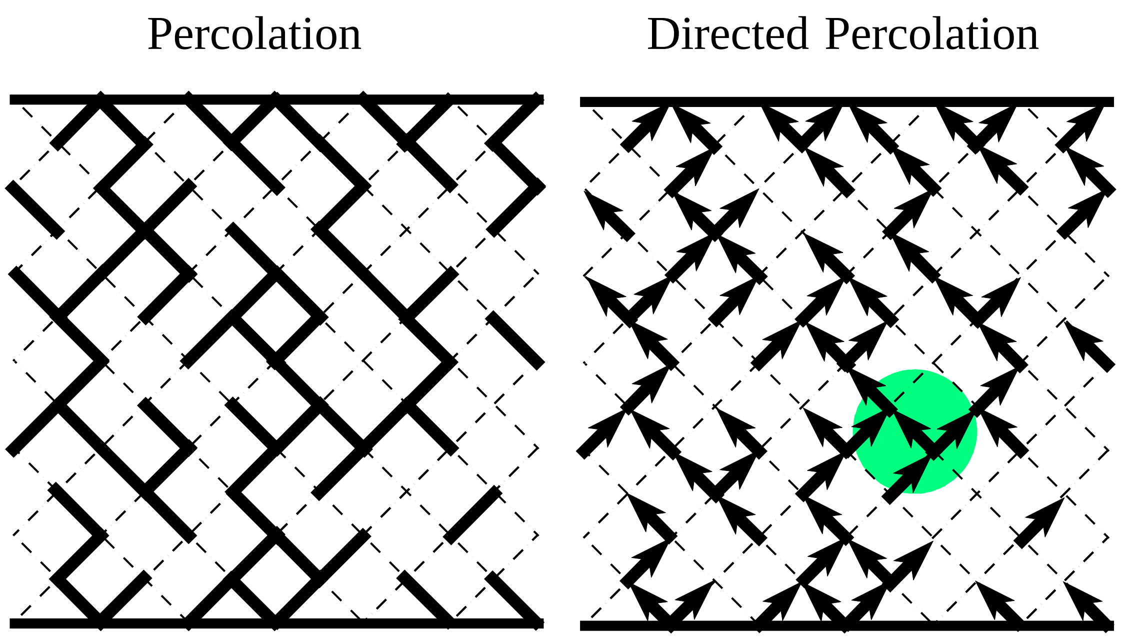
$\Rightarrow$ **Nonequilibrium phase transition** at $\lambda = \lambda_c$



Steady state density ρ of active sites for a two-dimensional contact process.

Contact process and directed percolation

- Percolation and directed percolation are purely geometric process displaying geometric phase transitions
- Contact process can be mapped on directed percolation by interpreting time as the restricted direction



Phase transition in the contact process falls into the directed percolation universality class

Disordered contact process

Quenched spatial disorder:

Healing and infection rates μ and λ vary randomly in space (from site to site)

How does quenched disorder change the properties of the phase transition?

Harris criterion:

- Critical point is stable against disorder if spatial correlation length exponent fulfills $\nu_\perp d > 2$
- Directed percolation universality class: Harris criterion violated for all $d < 4$

Properties of the dirty phase transition:

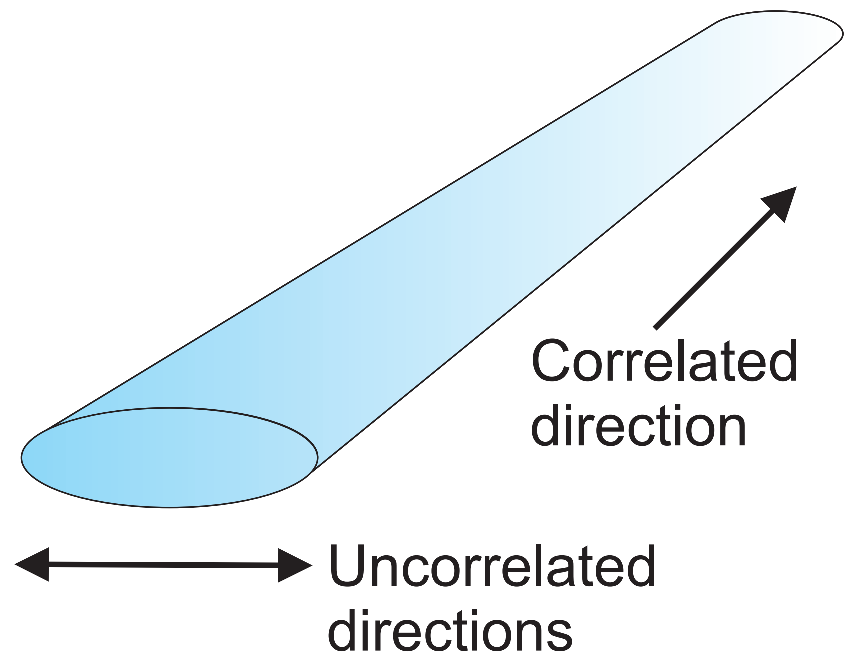
- Renormalization group (Janssen): run-away flow to large disorder
- Early MC simulations: violations of scaling, non-universal behavior
- Very recently (Hooyberghs et al): infinite randomness CP, activated scaling

Extended defects

Correlated disorder:
- impurities are not point-like but extended (linear, planar)
- correlations increase disorder effects ("harder to average out")

Rare regions:
- large spatial regions devoid of impurities
- can be locally in the active phase even if bulk is still inactive
- linear defects lead to rare regions extended in one dimension

$\Rightarrow$ rare region can undergo true phase transition independently from the bulk



Global phase transition is smeared by extended defects

Smeared nonequilibrium phase transition

Probability for finding a rare region: $w \sim \exp(-pL_r^{d_r})$

Critical point of rare region: $\lambda_c(L_r) - \lambda_c^0 = AL_r^{-\phi}$ $\quad(\phi = 1/\nu_\perp$ FSS shift exponent)

Total stationary density

sum of rare region contributions $\rho_{st}(\lambda) \sim \exp\left(-B(\lambda - \lambda_c^0)^{-d_r\nu_\perp}\right)$

$\Rightarrow$ exponential tail towards clean critical point

Time evolution of the density

each island has its own correlation time $\xi_t(\lambda, L_r)$

total time-dependent density $\rho(\lambda, t) \sim \int dL_r \ \exp\left[-\tilde{p}L_r^{d_r} - Dt/\xi_t(\Delta, L_r)\right]$

$\Rightarrow$ clean critical point: stretched exponential $\ln \rho(t) \sim -t^{d_r/(d_r+z)}$

$\Rightarrow$ tail of the smeared transition: power law $\rho(t) - \rho(\infty) \sim t^{-\psi}$

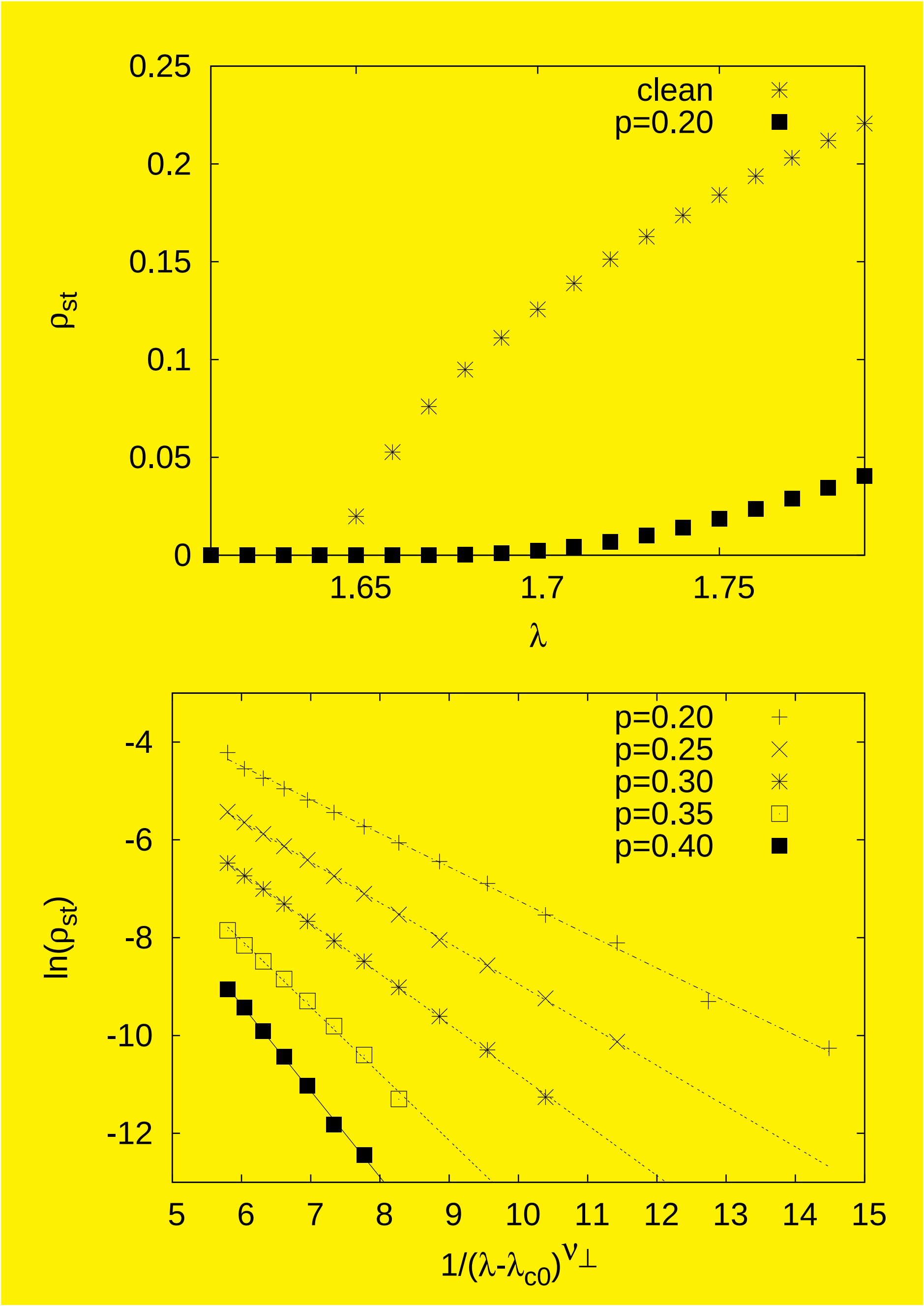
Monte-Carlo simulations

- two-dimensional contact process
- defects are introduced by making infection rate λ random
- binary distribution:
  weak sites of density p where λ is reduced by factor $c = 0.2$
- correlated disorder: defects consist of lines of weak sites

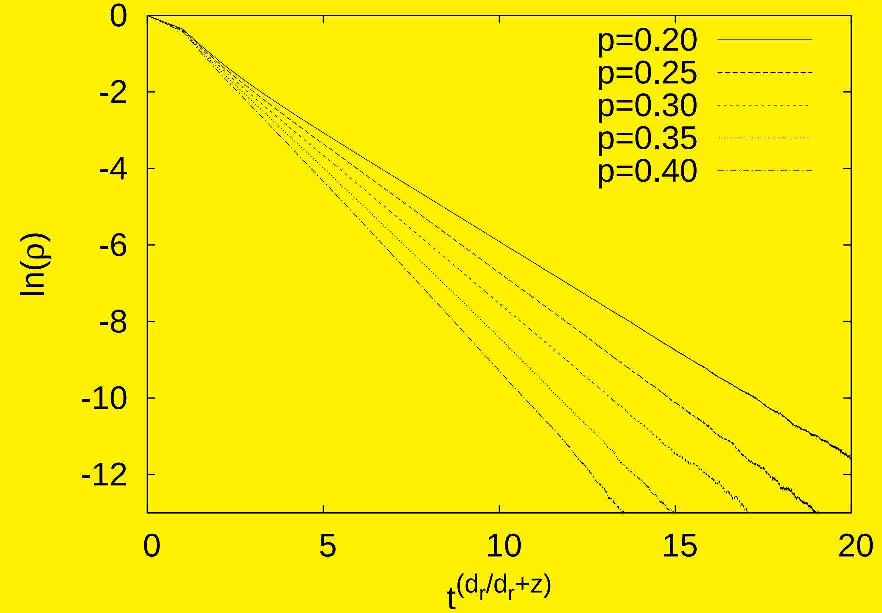
Steady state density ρ:

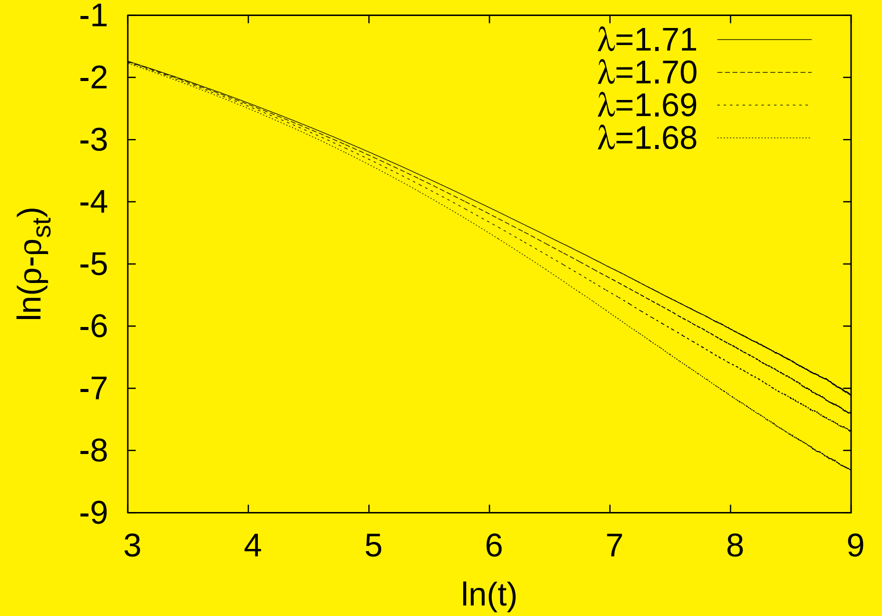
- ρ develops exponential tail towards the inactive phase
- functional form follows prediction from optimal fluctuation theory

$$\rho \sim \exp\left(-B(\lambda - \lambda_c^0)^{-d_r \nu_\perp}\right)$$



Monte-Carlo simulations





Time evolution of density:

- at the clean critical point $\lambda = \lambda_c^0$: density decays following stretched exponential

$$\ln \rho(t) \sim -t^{d_r/(d_r+z)}$$

- tail of smeared transition $\lambda > \lambda_c^0$: power-law approach to steady state density

$$\rho(t) - \rho(\infty) \sim t^{-\psi}$$

Conclusions

- directed percolation critical point is **unstable** against spatial disorder (Harris)
- spatial disorder **correlations increase** the disorder effects
- extended (linear or planar) defects destroy the sharp phase transition because **rare regions** can undergo the transition **independently** from the bulk
- active phase develops an exponential tail towards the clean critical point
- in this tail the system is very inhomogeneous
- smearing scenario is very general, it has also been found in classical and quantum equilibrium phase transitions

Extended defects completely destroy the phase transition in the contact process by smearing