# Quantum Griffiths effects in disordered Heisenberg magnets


**Thomas Vojta[1] and Jörg Schmalian[2]**

[1]Department of Physics, University of Missouri-Rolla
[2]Department of Physics, Iowa State University and DOE Ames Lab


- Phase transitions and disorder
- Rare regions and Griffiths singularities
- Disordered itinerant Heisenberg magnets

# Phase transitions and disorder

**Common lore:**

Harris criterion, condition for homogeneous, sharp transition: $d\nu > 2$

(spatial fluctuations of local $T_c(x)$ within correlation volume must be smaller than distance from global critical point $T_c$)

- if clean critical point fulfills Harris criterion, it is stable against weak disorder (inhomogeneities vanish at large length scales)

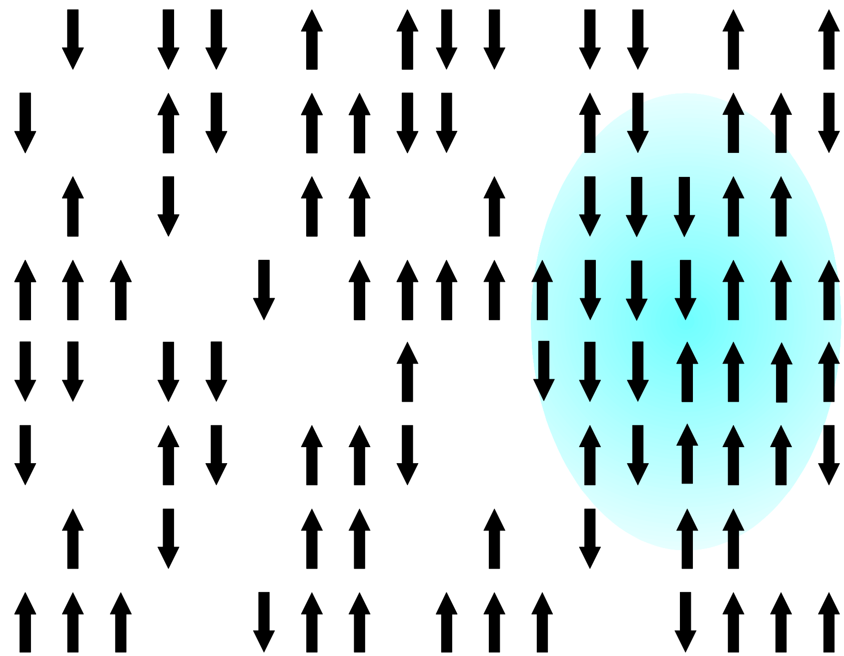
even if the clean critical point is unstable (Harris criterion violated), transition is generically sharp

- inhomogeneities remain finite at all length scales
  $\Rightarrow$ conventional finite-disorder critical point which fulfills $d\nu > 2$
  or
- inhomogeneities diverge under coarse graining
  $\Rightarrow$ infinite-randomness critical point

# Rare regions and Griffiths singularities

## Example: classical dilute ferromagnet

- critical temperature $T_c$ is reduced compared to clean value $T_{c0}$
- for $T_c < T < T_{c0}$:
  no global order but local order on rare, large islands devoid of impurities
- locally ordered islands have slow dynamics



probability for finding rare region of size $L$: $w \sim \exp(-pL^d)$
contribution of single rare region to free energy: $\Delta f \sim L^x$

$\Rightarrow$ singular free energy everywhere in the Griffiths region $(T_c < T < T_{c0})$

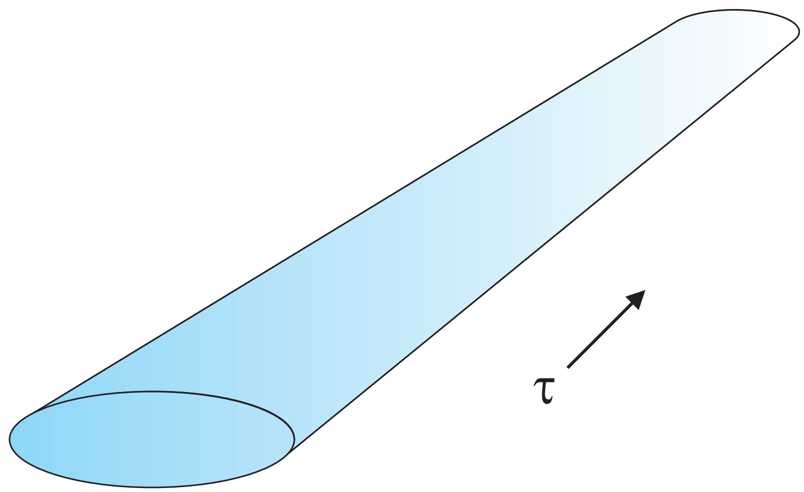
**Classical Griffiths singularities are generically weak, magnetic susceptibility remains finite in the Griffiths region**

# Disorder at quantum phase transitions

**quantum phase transitions occur at zero temperature**

- imaginary-time direction becomes important for critical fluctuations
- quenched disorder is totally correlated in time direction

$\Rightarrow$ **disorder effects are enhanced**




rare region at a quantum phase transition

**Quantum Griffiths effects**

- rare regions at a QPT are finite in space but infinite in imaginary time
- dynamics of the rare regions becomes even slower than in the classical case

$\Rightarrow$ **Griffiths singularities are enhanced**

# Disordered itinerant Heisenberg magnets

## antiferromagnetic quantum phase transition of itinerant electrons

LGW free energy functional

$$\Phi = T \sum_{q,\omega_n} \mathbf{M}(q,\omega_n) \left[ r_0 + q^2 + |\omega_n|^{2/z} \right] \mathbf{M}(-q,-\omega_n) + \frac{u}{2N} \int d^d x \, d\tau \mathbf{M}^4(x,\tau)$$

- localized quantum rotors: undamped dynamics, $z = 1$
- itinerant magnets: magnetic fluctuations are damped, $z = 2$
  $|\omega_n|$ corresponds to power-law interaction $\sim 1/(\tau - \tau')^2$ in imaginary time

## Quenched disorder:

energy gap becomes random function of position $r_0 \Longrightarrow r + \delta r(x)$

$\delta r$ is random variable with $\langle \delta r(x) \rangle = 0, \quad \langle \delta r(x) \delta r(y) \rangle = v \delta(x - y)$

# Quantum Griffiths effects: Scaling arguments

## Single rare region:

- equivalent to 1d Heisenberg model with $1/r^2$ interaction
- rare region is at its lower critical dimension $d_c^-$

$\Rightarrow$ energy gap vanishes exponentially with size $\epsilon \sim \exp(-bL^d)$

## Quantum Griffiths effects:

combine energy gap with probability for finding rare region $w \sim \exp(-pL^d)$

- power-law density of states $\rho(\epsilon) \sim \epsilon^{c/b-1} = \epsilon^{d/z'-1}$
- average local susceptibility $[\chi_{loc}(\tau)] \sim \tau^{-d/z'}$

$$[\chi_{loc}(T)] \sim \int_0^{1/T} d\tau [\chi_{loc}(\tau)] \sim T^{d/z'-1}$$

- RR contribution to specific heat $\Delta C \sim T^{d/z'}$

# Quantum Griffiths effects: Large-$N$ calculation

Large-$N$ limit: $u\mathbf{M}^4 \longrightarrow 2uN\,\mathbf{M}^2\,\langle\mathbf{M}^2\rangle$ ($N$: number of spin components)

**Self-consistent equation for the energy gap:**

$$\epsilon = r + u\langle\mathbf{M}^2\rangle = r + u\frac{T}{L^d}\sum_{q,\omega_n}\frac{1}{\epsilon + q^2 + |\omega_n|^{2/z}}$$

**Solutions for small $\epsilon$:**

$T \neq 0$: $q$ and $\omega_n$ sums are discrete, leading contribution from $q = \omega_n = 0$
$\epsilon \sim L^{-d} \longrightarrow$ **weak classical Griffiths effects**

$T = 0, z < 2$: frequency sum turns into integration, singularity gets weaker
$\epsilon \sim L^{-2d/(2-z)} \longrightarrow$ **Griffiths effects still exponentially weak**

$T = 0, z = 2$: marginal case, integral diverges only logarithmically
$\epsilon \sim L^{-2}\exp(-bL^d) \longrightarrow$ **power-law quantum Griffiths effects**

# Classification of dirty phase transitions according to importance of rare regions

| Dimensionality of rare regions | Griffiths effects | Dirty critical point | Examples (classical PT, QPT) |
|---|---|---|---|
| $d_{RR} < d_c^-$ | weak exponential | conv. finite disorder | class. magnet with point defects<br>dilute bilayer Heisenberg model |
| $d_{RR} = d_c^-$ | strong power-law | infinite randomness | Ising model with linear defects<br>random quantum Ising model<br>itin. quantum Heisenberg magnet? |
| $d_{RR} > d_c^-$ | RR become static | smeared transition | Ising model with planar defects<br>itinerant quantum Ising magnet |

# Conclusions

- even weak disorder can have surprisingly strong effects on a quantum phase transition
- **rare regions** play a much bigger role quantum phase transitions than a classical transitions
- **effective dimensionality** of rare regions determines **overall phenomenology** of phase transitions in disordered systems
- in itinerant **Heisenberg** magnets: rare regions lead to strong **power-law quantum** Griffiths effects
- in itinerant **Ising** magnets: sharp phase transition is destroyed by **smearing** because static order forms on rare spatial regions

**Quenched disorder at quantum phase transitions leads to a rich variety of new effects and exotic phenomena**