Itinerant ferromagnetic quantum phase transition

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- Coupling between magnetization and additional fermionic modes
  $\Rightarrow$ non-local order parameter theory
  $\Rightarrow$ clean electrons: 1st order transition
  $\Rightarrow$ dirty electrons: non-mean field exponents

- Coupled local field theory for all soft modes
  $\Rightarrow$ fluctuation induced transitions and log. corrections to scaling

Motivation

Itinerant ferromagnetic quantum phase transition is one of the most obvious quantum phase transitions

Experiments

- MnSi, UGe_2, $ZrZn_2$ – pressure tuned
- Ni_xPd_{1-x}, $URu_{2-x}Re_xSi_2$ – composition tuned

Transition can be

- first order (MnSi, UGe_2) or
- second order with mean-field exponents ($ZrZn_2$, Ni_xPd_{1-x}) or
- second order with non-mean-field exponents ($URu_{2-x}Re_xSi_2$)

Experiments seemingly inconclusive

Order parameter field theory of the ferromagnetic quantum phase transition

Starting point: microscopic model of interacting electrons

$$S = S_0 + S_{trip} = S_0 + \frac{\Gamma_t}{2} \int d^d r d\tau \; \mathbf{n}_s(\tau) \cdot \mathbf{n}_s(\tau)$$

$\mathbf{n}_s$ – spin density
S_0 – reference system, interacting Fermi liquid

Landau Ginzburg-Wilson philosophy

Derive an effective field theory in terms of the order parameter only, i.e., integrate out out other degrees of freedom

$\Rightarrow$ potentially dangerous if soft modes are integrated out !

Soft modes, long-range correlations, and generic scale invariance

long-range (power-law) correlations: due to soft (gapless) modes

critical phenomena: critical modes become soft at one particular point in the phase diagram

generic scale invariance: long-range correlations in large regions of the phase diagram due to additional soft modes in the system (due to conservation laws or broken symmetry)

Examples for generic scale invariance

- long-time tails in equilibrium correlation functions of classical fluids
- non-existence of virial expansion for transport coefficients
- long-range spatial correlations in classical non-equilibrium states
- weak localization effects in disordered electronic systems

Generic scale invariance in a 3D Fermi liquid

quasi-particle dispersion: $\Delta\epsilon(p) \sim |p - p_F|^3 \log |p - p_F|$
specific heat: $c_V(T) \sim T^3 \log T$

static spin susceptibility: $\chi^{(2)}(\mathbf{q}) = \chi^{(2)}(0) + c_3|\mathbf{q}|^2 \log \frac{1}{|\mathbf{q}|} + O(|\mathbf{q}|^2)$
in real space: $\chi^{(2)}(\mathbf{r} - \mathbf{r}') \sim |\mathbf{r} - \mathbf{r}'|^{-5}$

in general dimension: $\chi^{(2)}(\mathbf{q}) = \chi^{(2)}(0) + c_d|\mathbf{q}|^{d-1} + O(|\mathbf{q}|^2)$

Long-range correlations due to coupling to soft particle-hole excitations

For finite temperatures or finite magnetization particle-hole excitations are not soft $\Rightarrow$ singularities are cut-off:

finite magnetization/field: $|\mathbf{q}|^{d-1} \rightarrow (|\mathbf{q}| + m)^{d-1}$
finite temperatures: $|\mathbf{q}|^{d-1} \rightarrow (|\mathbf{q}| + T)^{d-1}$

Landau-Ginzburg-Wilson free energy functional

$$\Phi(\mathbf{M}) = \int dx dy\ \mathbf{M}(x) \left[1 - \Gamma_t \chi^{(2)}\right] \mathbf{M}(y)\ -\ \sum_{n=3}^{\infty} \frac{(-1)^n}{n!} \Gamma_t^{n/2} \int \chi^{(n)} \mathbf{M}^n$$

$\chi^{(n)}$ – n-point spin density correlation functions of reference system S_0

Fermi liquid:

$$\chi^{(2)}(\mathbf{q},\omega) = \text{const} + |\mathbf{q}|^{d-1} + \frac{|\omega|}{|\mathbf{q}|} \quad (d<3)$$

$$\chi^{(2)}(\mathbf{q},\omega) = \text{const} + |\mathbf{q}|^{2} \ln\frac{1}{|\mathbf{q}|} + \frac{|\omega|}{|\mathbf{q}|} \quad (d=3)$$

$$\chi^{(n)}(\mathbf{q},0) \sim |\mathbf{q}|^{d+1-n}$$

$\Rightarrow$ Fermi liquid singularities provide leading $\mathbf{q}$ dependence at QPT

Mechanism is very general, leads to singular LGW theory for all QPT with zero wavenumber order parameter

Resulting Landau-Ginzburg-Wilson functional

$$\Phi(\mathbf{M}) = \sum_{\mathbf{q},\omega} \mathbf{M}^*(\mathbf{q},\omega) \left[t + c_d|\mathbf{q}|^{d-1} + |\mathbf{q}|^2 + \frac{|\omega|}{|\mathbf{q}|} \right] \mathbf{M}(\mathbf{q},\omega)$$
$$+ \int u^{(4)}(\mathbf{q},\omega)\ \mathbf{M}^4\ + \ldots$$

- singular $|\mathbf{q}|^{d-1}$ term is leading for $d \leq 3$
- higher order coefficients $u^{(4)}(\mathbf{q},\omega)$ contain stronger singularities
- $|\mathbf{q}|^{d-1}$ term has unusual **negative sign**, $c_d < 0$
  $\Rightarrow$ continuous ferromagnetic transition seems to be impossible

Order parameter field theory for dirty electrons

Dirty Fermi liquid singularities:

$$\chi^{(2)}(\mathbf{q},\omega) = \text{const} - |\mathbf{q}|^{d-2} + \frac{|\omega|}{|\mathbf{q}|^2}$$
$$\chi^{(n)}(\mathbf{q},0) \sim |\mathbf{q}|^{d+2-2n}$$

electrons are diffusive ($\omega \sim q^2$) rather than ballistic ($\omega \sim q$)

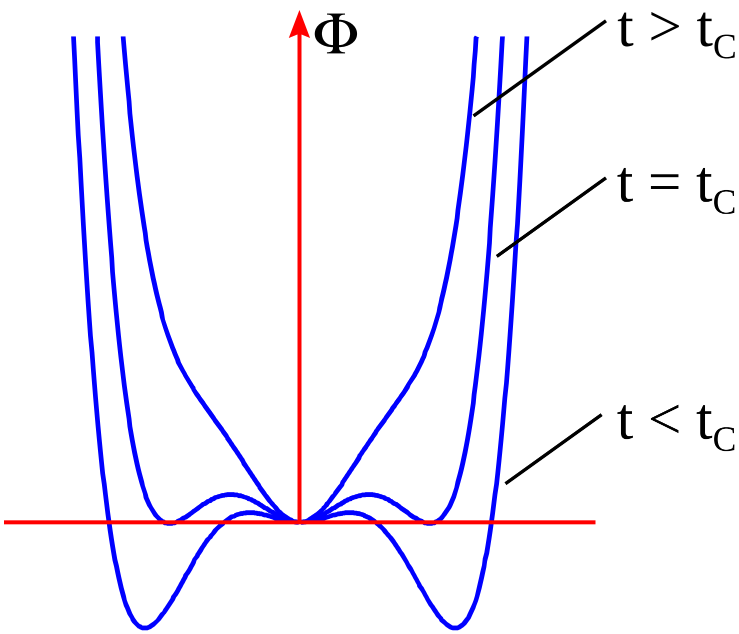
$$\Phi(\mathbf{M}) = \sum_{\mathbf{q},\omega} \mathbf{M}^*(\mathbf{q},\omega) \left[t + c_d |\mathbf{q}|^{d-2} + |\mathbf{q}|^2 + \frac{|\omega|}{|\mathbf{q}|^2} \right] \mathbf{M}(\mathbf{q},\omega) + O(\mathbf{M}^4)$$

- generic sign of the singular term is ferromagnetic, $c_d > 0$
  $\Rightarrow$ ferromagnetic QPT will be continuous
- power counting: Gaussian fixed point is stable
- non-mean field critical exponents (below $d = 4$):
  $z = \delta = d$, $\nu = 1/(d-2)$, $\eta = 4 - d$, $\beta = 2/(2-d)$, $\gamma = 1$

1st order transition in clean ferromagnets

Generalized mean-field theory, taking the cut-off singularities into account

$$\Phi = tm^2 - vm^4 \log(1/m) + um^4$$

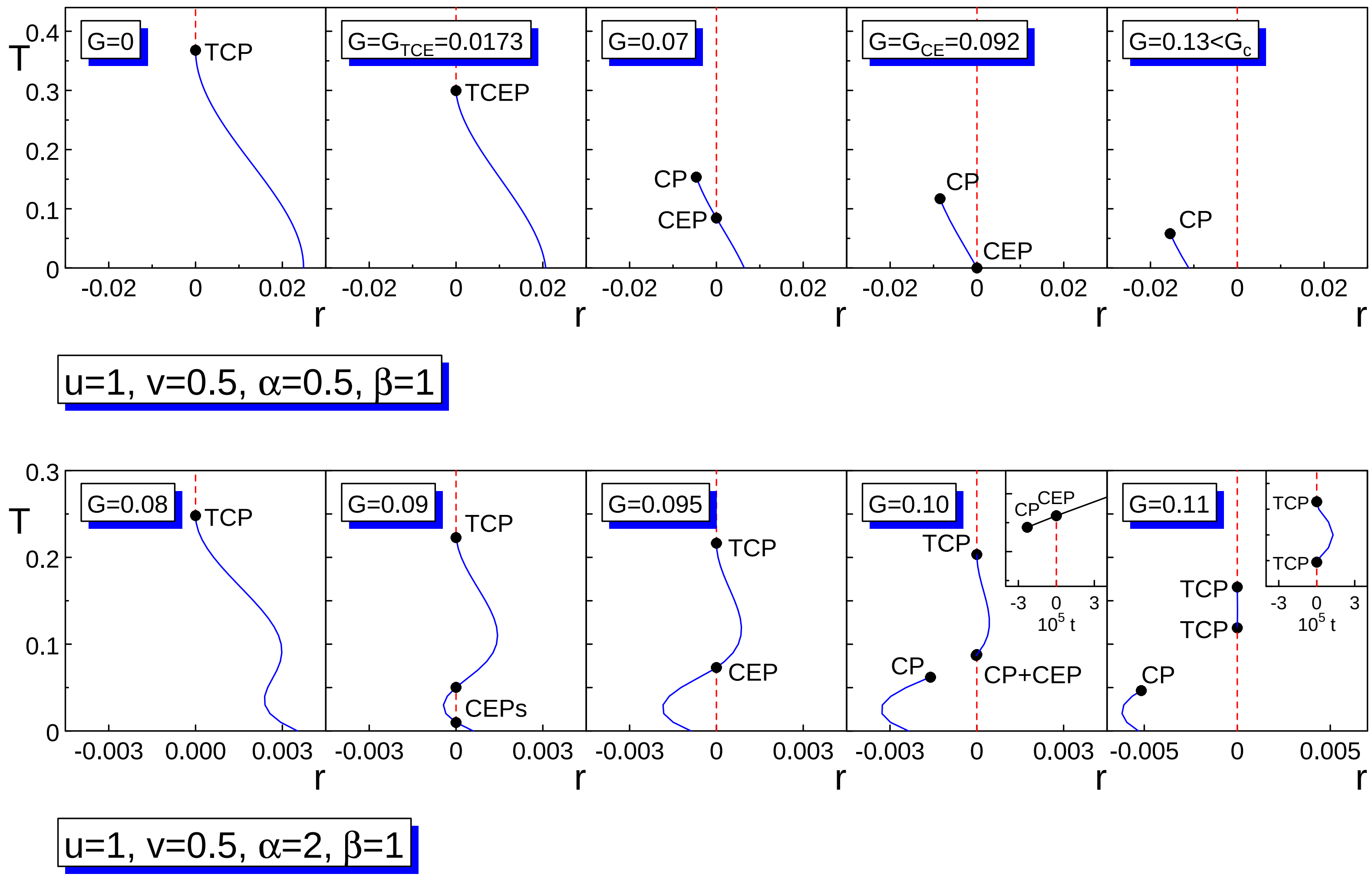


1st order transition because of negative sign of singular term

Power counting:
higher order terms irrelevant
mean-field theory essentially exact

Multicritical points and complicated phase diagrams

Including the effects of disorder and the various cut-offs leads to more sophisticated generalized mean-field theory



Coupled field theory

- order parameter theory not very satisfying conceptually
  $\Rightarrow$ all soft modes should be treated on the same footing
- power counting can be dangerous in a theory with singular vertices
- system has different time scales for critical fluctuations and fermionic soft modes

$\Rightarrow$ Coupled field theory for magnetization and fermionic soft modes

$$S[\mathbf{M}, q] = S_{\mathbf{M}}[\mathbf{M}] + S_q[q] + S_c[\mathbf{M}, q]$$

$S_{\mathbf{M}}[\mathbf{M}]$	local static magnetic LGW functional
$S_q[q]$	action of the soft fermionic particle-hole excitations dirty case: well-known NLσM (Wegner) clean case: clean analog of the NLσM
$S_c[\mathbf{M}, q]$	coupling, terms of the form $\mathbf{M}q, \mathbf{M}q^2, \ldots$

Results from the coupled field theory

Clean Ferromagnets

- 1st order transition found in non-local LGW theory is induced by fluctuations due to the fermionic soft modes
- if this transition is only weakly 1st order it can be destroyed by order parameter fluctuations (which feed back into the fermionic modes)
- resulting fluctuation induced 2nd order transition has an upper critical dimension of $d_c^+ = 3$.

Dirty Ferromagnets

- fixed point found in non-local LGW theory is marginally unstable (coupling $\mathbf{M}q^2$ is marginal)
- critical exponents remain unchanged (strongly non-mean field), but complicated log. corrections to scaling arise

Conclusions

- coupling between the order parameter and soft fermionic particle-hole excitations has drastic influence on the itinerant ferromagnetic quantum phase transition
- **clean electrons**
  - transition can be of first order (MnSi, UGe_2) or
  - second order with mean-field exponents in 3D ($ZrZn_2$, Ni_xPd_{1-x})
- **disordered electrons**
  - transition generically of 2nd order
  - non-mean field exponents ($URu_{2-x}Re_xSi_2$)

Ferromagnetic QPT is much more interesting than previously thought