Itinerant ferromagnetic quantum phase transition

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- Coupling between magnetization and additional fermionic modes
  ⇒ non-local order parameter theory
  ⇒ clean electrons: 1st order transition
  ⇒ dirty electrons: non-mean field exponents

- Coupled local field theory for all soft modes
  ⇒ fluctuation induced transitions and log. corrections to scaling
Motivation

Itinerant ferromagnetic quantum phase transition is one of the most obvious quantum phase transitions

Experiments

- \text{MnSi, UGe}_2, \text{ZrZn}_2 – pressure tuned
- \text{Ni}_x\text{Pd}_{1-x}, \text{URu}_{2-x}\text{Re}_x\text{Si}_2 – composition tuned

Transition can be

- first order (MnSi, UGe$_2$) or
- second order with mean-field exponents (ZrZn$_2$, Ni$_x$Pd$_{1-x}$) or
- second order with non-mean-field exponents (URu$_{2-x}$Re$_x$Si$_2$)

Experiments seemingly inconclusive
Order parameter field theory of the ferromagnetic quantum phase transition

Starting point: microscopic model of interacting electrons

\[ S = S_0 + S_{trip} = S_0 + \frac{\Gamma_t}{2} \int d^d r d\tau \ n_s(\tau) \cdot n_s(\tau) \]

\( n_s \) – spin density
\( S_0 \) – reference system, interacting Fermi liquid

Landau Ginzburg-Wilson philosophy

Derive an effective field theory in terms of the order parameter only, i.e., integrate out other degrees of freedom

\[ \Rightarrow \text{potentially dangerous if soft modes are integrated out!} \]
Soft modes, long-range correlations, and generic scale invariance

long-range (power-law) correlations: due to soft (gapless) modes

**critical phenomena:** critical modes become soft at one particular point in the phase diagram

**generic scale invariance:** long-range correlations in large regions of the phase diagram due to additional soft modes in the system (due to conservation laws or broken symmetry)

Examples for generic scale invariance

- long-time tails in equilibrium correlation functions of classical fluids
- non-existence of virial expansion for transport coefficients
- long-range spatial correlations in classical non-equilibrium states
- weak localization effects in disordered electronic systems
Generic scale invariance in a 3D Fermi liquid

quasi-particle dispersion: \[ \Delta \epsilon(p) \sim |p - p_F|^3 \log |p - p_F| \]
specific heat: \[ c_V(T) \sim T^3 \log T \]
static spin susceptibility: \[ \chi^{(2)}(\mathbf{q}) = \chi^{(2)}(0) + c_3 |\mathbf{q}|^2 \log \frac{1}{|\mathbf{q}|} + O(|\mathbf{q}|^2) \]
in real space: \[ \chi^{(2)}(\mathbf{r} - \mathbf{r}') \sim |\mathbf{r} - \mathbf{r}'|^{-5} \]
in general dimension: \[ \chi^{(2)}(\mathbf{q}) = \chi^{(2)}(0) + c_d |\mathbf{q}|^{d-1} + O(|\mathbf{q}|^2) \]

Long-range correlations due to coupling to soft particle-hole excitations

For finite temperatures or finite magnetization particle-hole excitations are not soft \( \Rightarrow \) singularities are cut-off:

finite magnetization/field: \[ |\mathbf{q}|^{d-1} \to (|\mathbf{q}| + m)^{d-1} \]
finite temperatures: \[ |\mathbf{q}|^{d-1} \to (|\mathbf{q}| + T)^{d-1} \]
Landau-Ginzburg-Wilson free energy functional

\[ \Phi(M) = \int dx dy \, M(x) \left[ 1 - \Gamma_t \chi^{(2)} \right] M(y) - \sum_{n=3}^{\infty} \frac{(-1)^n}{n!} \Gamma_t^{n/2} \int \chi^{(n)} M^n \]

\( \chi^{(n)} \) — \( n \)-point spin density correlation functions of reference system \( S_0 \)

**Fermi liquid:**

\[ \chi^{(2)}(q, \omega) = \text{const} + |q|^{d-1} + \frac{\omega}{|q|} \quad (d < 3) \]

\[ \chi^{(2)}(q, \omega) = \text{const} + |q|^2 \ln \frac{1}{|q|} + \frac{\omega}{|q|} \quad (d = 3) \]

\[ \chi^{(n)}(q, 0) \sim |q|^{d+1-n} \]

\( \Rightarrow \) Fermi liquid singularities provide leading \( q \) dependence at QPT

Mechanism is very general, leads to singular LGW theory for all QPT with zero wavenumber order parameter
Resulting Landau-Ginzburg-Wilson functional

\[
\Phi(M) = \sum_{q, \omega} M^*(q, \omega) \left[ t + c_d |q|^{d-1} + |q|^2 + \frac{\omega}{|q|} \right] M(q, \omega) \\
+ \int u^{(4)}(q, \omega) M^4 + ...
\]

- singular $|q|^{d-1}$ term is leading for $d \leq 3$
- higher order coefficients $u^{(4)}(q, \omega)$ contain stronger singularities
- $|q|^{d-1}$ term has unusual negative sign, $c_d < 0$
  \(\Rightarrow\) continuous ferromagnetic transition seems to be impossible
Order parameter field theory for dirty electrons

Dirty Fermi liquid singularities:

\[
\chi^{(2)}(q, \omega) = \text{const} - |q|^{d-2} + \frac{\omega}{|q|^2}
\]

\[
\chi^{(n)}(q, 0) \sim |q|^{d+2-2n}
\]

electrons are diffusive \((\omega \sim q^2)\) rather than ballistic \((\omega \sim q)\)

\[
\Phi(M) = \sum_{q, \omega} M^*(q, \omega) \left[ t + c_d |q|^{d-2} + |q|^2 + \frac{\omega}{|q|^2} \right] M(q, \omega) + O(M^4)
\]

- generic sign of the singular term is ferromagnetic, \(c_d > 0\)
  \(\Rightarrow\) ferromagnetic QPT will be continuous

- power counting: Gaussian fixed point is stable

- non-mean field critical exponents (below \(d = 4\)):
  \(z = \delta = d, \ \nu = 1/(d-2), \ \eta = 4 - d, \ \beta = 2/(2 - d), \ \gamma = 1\)
1st order transition in clean ferromagnets

Generalized mean-field theory, taking the cut-off singularities into account

$$\Phi = tm^2 - vm^4 \log(1/m) + um^4$$

1st order transition because of negative sign of singular term

**Power counting:**
higher order terms irrelevant
mean-field theory essentially exact
Multicritical points and complicated phase diagrams

Including the effects of disorder and the various cut-offs leads to more sophisticated generalized mean-field theory.

\[ u=1, \, v=0.5, \, \alpha=0.5, \, \beta=1 \]

\[ u=1, \, v=0.5, \, \alpha=2, \, \beta=1 \]
Coupled field theory

- order parameter theory not very satisfying conceptually
  ⇒ all soft modes should be treated on the same footing
- power counting can be dangerous in a theory with singular vertices
- system has different time scales for critical fluctuations and fermionic soft modes

⇒ Coupled field theory for magnetization and fermionic soft modes

\[ S[M, q] = S_{M}[M] + S_{q}[q] + S_{c}[M, q] \]

- \( S_{M}[M] \) local static magnetic LGW functional
- \( S_{q}[q] \) action of the soft fermionic particle-hole excitations
dirty case: well-known \( \text{NL}\sigma\text{M} \) (Wegner)
clean case: clean analog of the \( \text{NL}\sigma\text{M} \)
- \( S_{c}[M, q] \) coupling, terms of the form \( Mq, Mq^2, \ldots \)
Results from the coupled field theory

Clean Ferromagnets

- 1st order transition found in non-local LGW theory is induced by fluctuations due to the fermionic soft modes
- if this transition is only weakly 1st order it can be destroyed by order parameter fluctuations (which feed back into the fermionic modes)
- resulting fluctuation induced 2nd order transition has an upper critical dimension of \( d^c_+ = 3 \).

Dirty Ferromagnets

- fixed point found in non-local LGW theory is marginally unstable (coupling \( Mq^2 \) is marginal)
- critical exponents remain unchanged (strongly non-mean field), but complicated log. corrections to scaling arise
Conclusions

• coupling between the order parameter and soft fermionic particle-hole excitations has drastic influence on the itinerant ferromagnetic quantum phase transition

• clean electrons
  – transition can be of first order (MnSi, UGe$_2$) or
  – second order with mean-field exponents in 3D (ZrZn$_2$, Ni$_x$Pd$_{1-x}$)

• disordered electrons
  – transition generically of 2nd order
  – non-mean field exponents (URu$_{2-x}$Re$_x$Si$_2$)

Ferromagnetic QPT is much more interesting than previously thought