Infinite-randomness quantum critical points induced by dissipation

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- Motivation: superconducting nanowires and itinerant quantum magnets
- Strong-disorder renormalization group
- Infinite-randomness quantum critical point

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- Strong-disorder renormalization group
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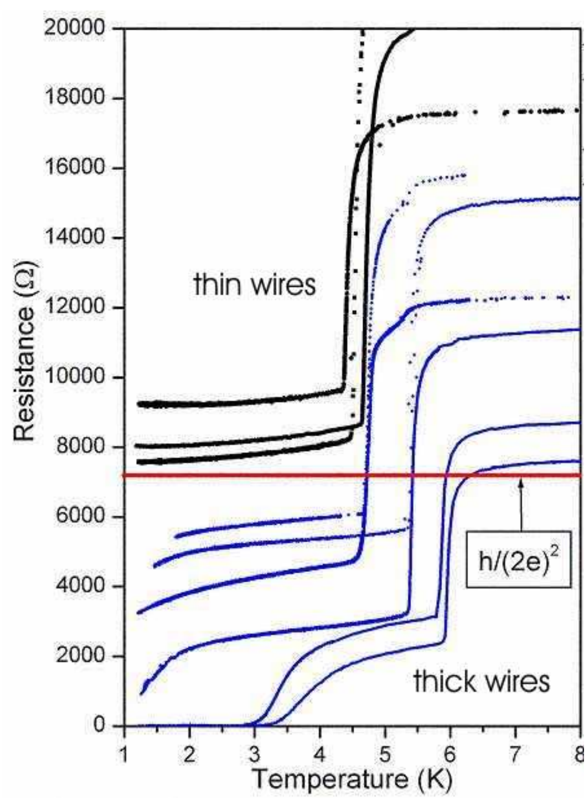
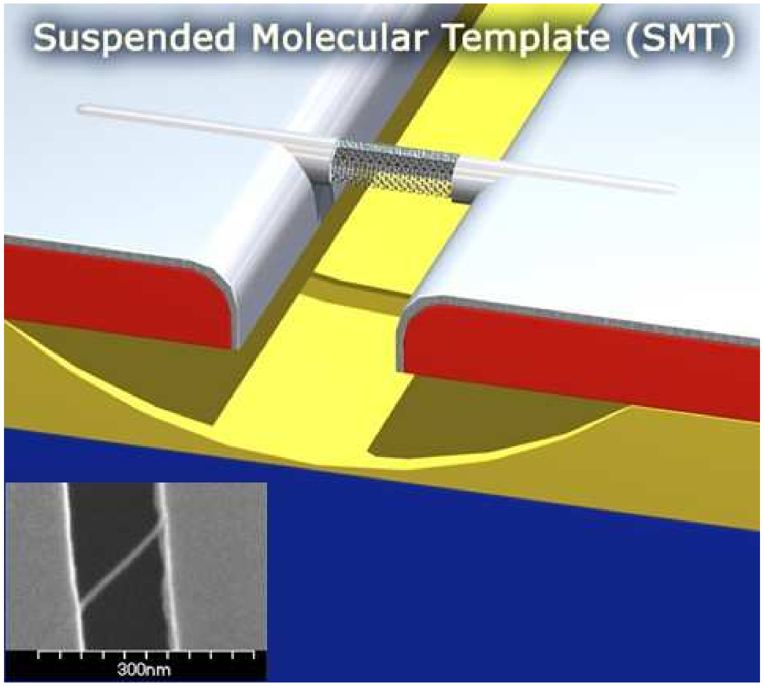
Experiment I: Superconductivity in ultrathin nanowires

- ultrathin MoGe wires (width $\sim$ 10 nm)
- produced by molecular templating using a single carbon nanotube
  [A. Bezryadin et al., Nature 404, 971 (2000)]

superconductor-metal QPT as function of wire thickness



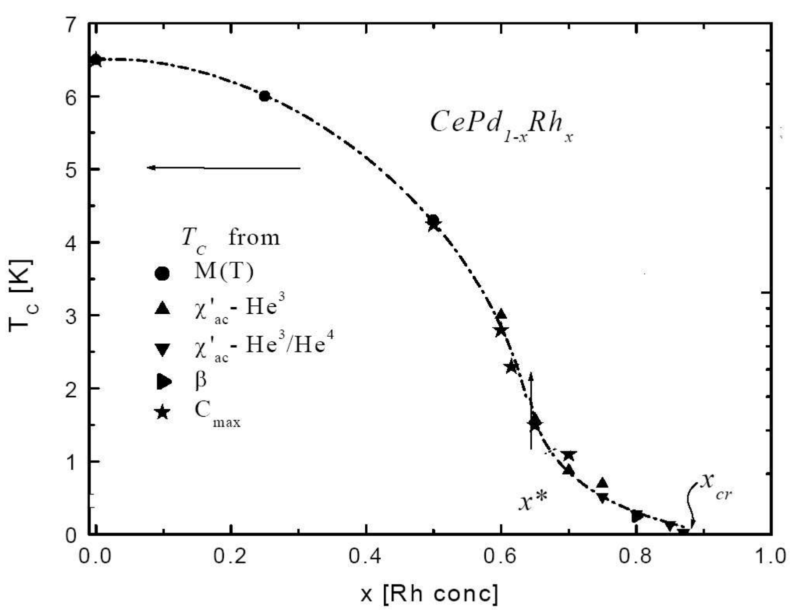


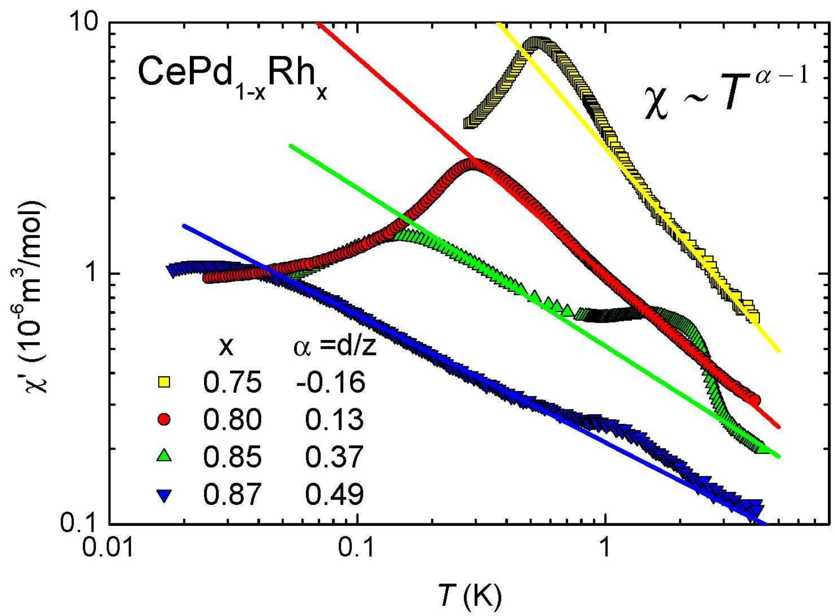
Pair breaking mechanism:

- magnetic impurities at the surface
  $\Rightarrow$ quenched **disorder**
- gapless excitations in metal phase
  $\Rightarrow$ Ohmic **dissipation**

Experiment II: Itinerant quantum magnets

- quantum phase transitions between paramagnetic metal and ferromagnetic or antiferromagnetic metal
- transition often controlled by chemical composition ⇒ **disorder** appears naturally
- magnetic modes damped due to coupling to fermions ⇒ Ohmic **dissipation**
- typical example: ferromagnetic transition in $\mathrm{CePd}_{1-x}\mathrm{Rh}_x$





(Sereni et al., PRB **75** (2007) 024432 + Westerkamp et al., PRL **102** (2009) 206404)

What is the fate of a quantum phase transition under the combined influence of disorder and dissipation?

- Motivation: superconducting nanowires and itinerant quantum magnets
- **Strong-disorder renormalization group**
- Infinite-randomness quantum critical point

Dissipative $O(N)$ order parameter field theory

N-component ($N > 1$) order parameter field $\varphi(\mathbf{x},\tau)$ in d dimensions derived by standard methods (Hubbard-Stratonovich transformation etc.)

$$S = T \sum_{\mathbf{q},\omega_n} \left(r + \xi_0^2 \mathbf{q}^2 + \gamma\,|\omega_n|\right) |\varphi(\mathbf{q},\omega_n)|^2 + \frac{u}{2N} \int \mathrm{d}^d x \mathrm{d}\tau \; \varphi^4(\mathbf{x},\tau)$$

Disorder: $\left\{ \begin{array}{c} \text{distance } r \text{ from criticality} \\ \text{bare correlation length } \xi_0 \\ \text{Ohmic dissipation constant } \gamma \end{array} \right\}$ random functions of position

- Superconductor-metal quantum phase transition in nanowires ($d = 1, N = 2$)
  $\varphi(\mathbf{x},\tau)$ represents local Cooper pair operator (Sachdev, Werner, Troyer 2004)
- Hertz' theory of itinerant quantum Heisenberg antiferromagnets ($d = 3, N = 3$)
  $\varphi(\mathbf{x},\tau)$ represents staggered magnetization (Hertz 1976)

Strong-disorder renormalization group

- introduced by Ma, Dasgupta, Hu (1979), further developed by Fisher (1992, 1995)
- asymptotically exact if disorder distribution becomes broad under RG

Basic idea: Successively integrate out the local high-energy modes and renormalize the remaining degrees of freedom.

discretized order-parameter field theory for "rotor" variables $\phi_i(\tau)$

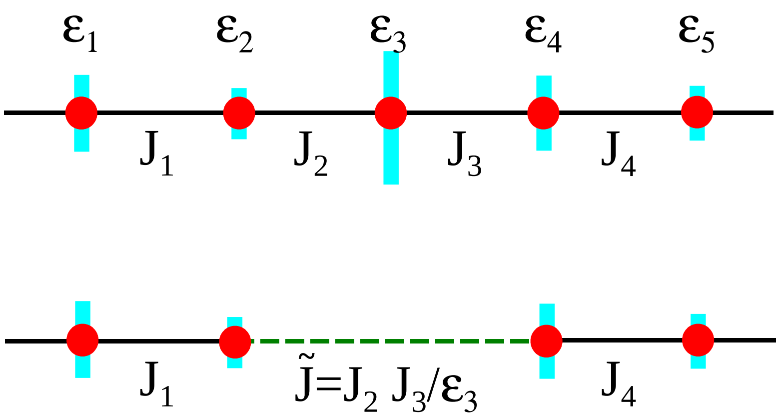
$$S = T \sum_{i,\omega_n} (\epsilon_i + \gamma_i |\omega_n|) \, |\phi_i(\omega_n)|^2 - T \sum_{i,\omega_n} J_i \, \phi_i(-\omega_n) \, \phi_{i+1}(\omega_n)$$

the competing local energies are:

- interactions (bonds) J_i favoring the ordered phase
- local "gaps" ϵ_i favoring the disordered phase

$\Rightarrow$ in each RG step, integrate out largest among all J_i and ϵ_i

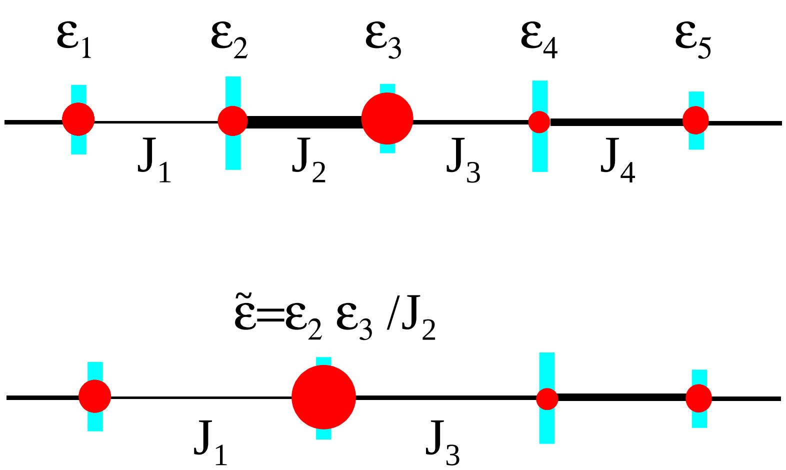
Recursion relations in one dimension



if largest energy is a gap, e.g., $\epsilon_3 \gg J_2, J_3$**:**

- site 3 is removed from the system
- coupling to neighbors is treated in 2nd order perturbation theory

new renormalized bond $\tilde{J} = J_2 J_3/\epsilon_3$



if largest energy is a bond, e.g., $J_2 \gg \epsilon_2, \epsilon_3$**:**

- rotors of sites 2 and 3 are parallel
- can be replaced by single rotor with moment $\tilde{\mu} = \mu_2 + \mu_3$

renormalized gap $\tilde{\epsilon} = \epsilon_2 \epsilon_3 / J_2$

Renormalization-group flow equations

- strong disorder RG step is iterated, gradually reducing maximum energy Ω
- competition between cluster aggregation and decimation
- leads to larger and larger clusters connected by weaker and weaker bonds

$\Rightarrow$ **flow equations** for the full probability distributions $P(J)$ and $R(\epsilon)$

$$
\begin{aligned}
-\frac{\partial P}{\partial \Omega} &= [P(\Omega) - R(\Omega)]\, P + R(\Omega) \int dJ_1 dJ_2\, P(J_1) P(J_2)\, \delta\left(J - \frac{J_1 J_2}{\Omega}\right) \\
-\frac{\partial R}{\partial \Omega} &= [R(\Omega) - P(\Omega)]\, R + P(\Omega) \int d\epsilon_1 d\epsilon_2\, R(\epsilon_1) R(\epsilon_2)\, \delta\left(\epsilon - \frac{\epsilon_1 \epsilon_2}{\Omega}\right)
\end{aligned}
$$

Flow equations are identical to those of the **random transverse-field Ising chain**

Note symmetry between J and ϵ!

Fixed points

If bare distributions do not overlap:

$\langle \ln \epsilon \rangle > \langle \ln J \rangle$: no clusters formed – disordered phase

$\langle \ln \epsilon \rangle < \langle \ln J \rangle$: all sites connected – ordered phase

If bare distributions do overlap:

$\langle \ln \epsilon \rangle > \langle \ln J \rangle$: rare clusters – disordered Griffiths phase

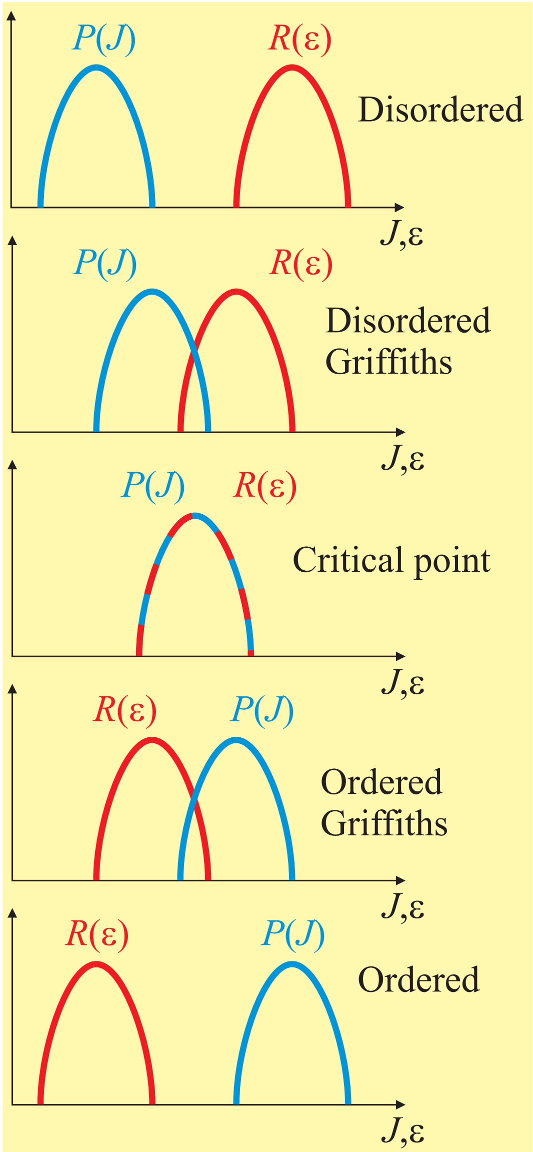
$\langle \ln \epsilon \rangle < \langle \ln J \rangle$: rare "holes" – ordered Griffiths phase

$\langle \ln \epsilon \rangle = \langle \ln J \rangle$: cluster aggregation and decimation balance at all energies – **critical point**

$$\mathcal{P}(\zeta) = \frac{1}{\Gamma} e^{-\zeta/\Gamma}, \quad \mathcal{R}(\beta) = \frac{1}{\Gamma} e^{-\beta/\Gamma}$$

log. variables $\zeta = \ln(\Omega/J)$, $\beta = \ln(\Omega/\epsilon)$, $\Gamma = \ln(\Omega_0/\Omega)$

Distributions become infinitely broad at critical point

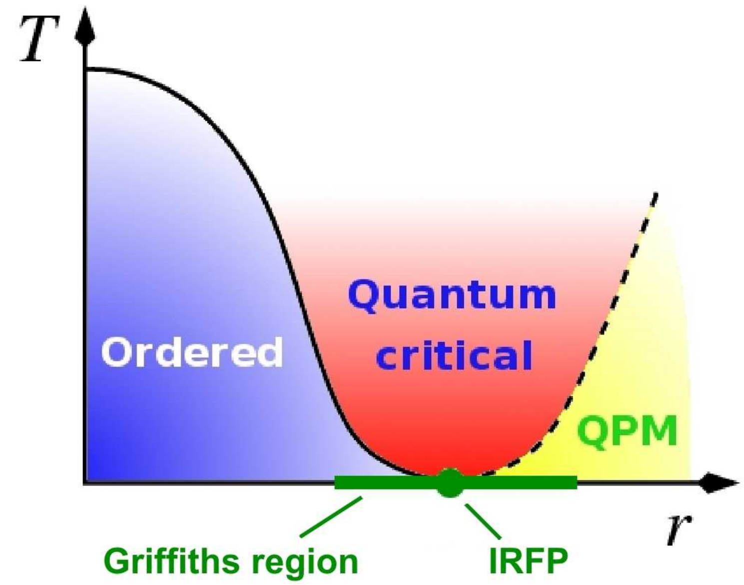


initial (bare) distributions

- Motivation: superconducting nanowires and itinerant quantum magnets
- Strong-disorder renormalization group
- **Infinite-randomness quantum critical point**

Critical behavior

- at critical FP, disorder scales to ∞
  $\Rightarrow$ **infinite-randomness critical point**
- activated dynamical scaling $\ln(1/\Omega) \sim L^\psi$ with tunneling exponent $\psi = 1/2$
- moments of surviving clusters grow like $\mu \sim \ln^\phi(1/\Omega)$ with $\phi = (1+\sqrt{5})/2$
- average correlation length diverges as $\xi \sim |r|^{-\nu}$ with $\nu = 2$



dissipative $O(N)$ order parameter is in universality class of **dissipationless** random transverse-field Ising model.

finite-temperature phase boundary and crossover line take unusual form

$$T_c \sim \exp(-\text{const}\,|r|^{-\nu\psi})$$

Quantum Griffiths regions:

- power-law dynamical scaling with nonuniversal exponent

Critical thermodynamics

Order parameter susceptibility and specific heat:
run RG down to energy scale $\Omega = T$ and consider remaining clusters as free

$$\chi(r,T) = \frac{1}{T}\left[\ln(1/T)\right]^{2\phi-d/\psi}\Theta_\chi\left(r^{\nu\psi}\ln(1/T)\right)$$

$$C(r,T) = \left[\ln(1/T)\right]^{-d/\psi}\Theta_C\left(r^{\nu\psi}\ln(1/T)\right)$$

at criticality: $\chi \sim \frac{1}{T}\left[\ln(1/T)\right]^{2\phi-d/\psi}$, in Griffiths phase: $\chi \sim T^{d/z'-1}$

Dynamic susceptibilities at $T=0$:
found by running RG to energy scale $\Omega \approx \omega$

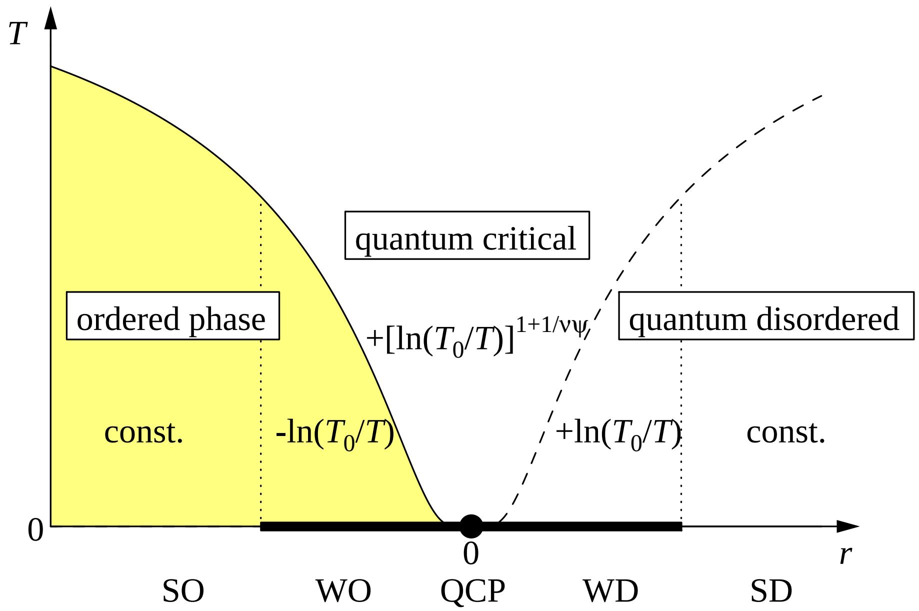
$$\mathrm{Im}\chi(r,\omega) \sim \frac{1}{\omega}\left[\ln(1/\omega)\right]^{\phi-d/\psi}\ X\left(r^{\nu\psi}\ln(1/\omega)\right)$$

$$\mathrm{Im}\chi^{\mathrm{loc}}(r,\omega) \sim \frac{1}{\omega}\left[\ln(1/\omega)\right]^{-d/\psi}\ X^{\mathrm{loc}}\left(r^{\nu\psi}\ln(1/\omega)\right)$$

Grüneisen parameter

Grüneisen parameter: ratio between thermal expansion coefficient and specific heat

$$\Gamma = \frac{\beta}{c_p} = -\frac{(\partial S/\partial p)_T}{V_m T (\partial S/\partial T)_p}$$



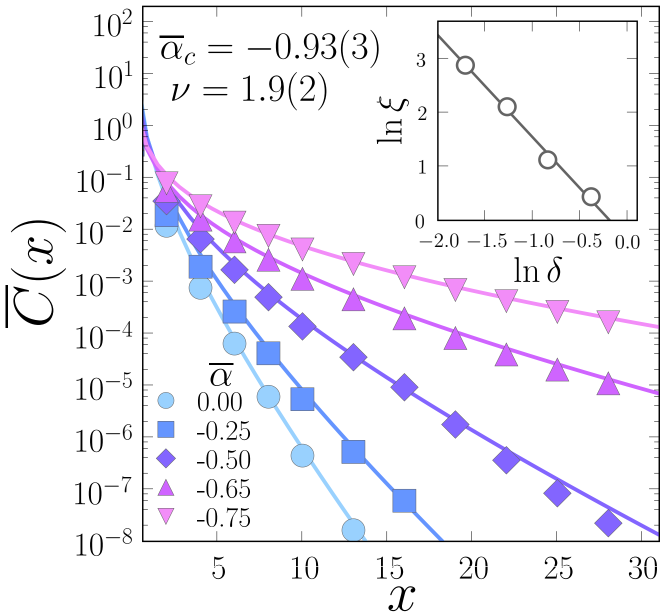
- at criticality:

$$\Gamma = -\frac{\psi}{V_m p_c d}\,\frac{\Phi'(0)}{\Phi(0)}\,[\ln(T_0/T)]^{1+1/(\nu\psi)}$$

- in the Griffiths phase:

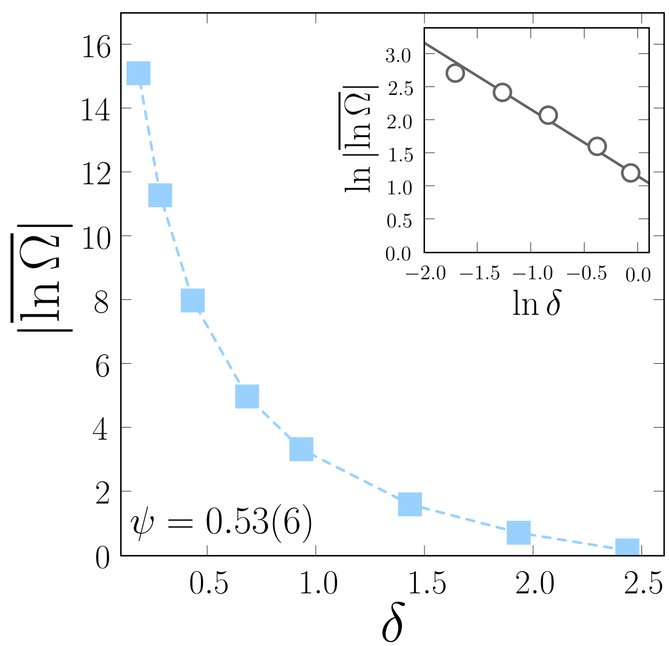
$$\Gamma = \frac{1}{V_m}\,\frac{\nu\psi}{p-p_c}\,\ln(T_0/T)$$

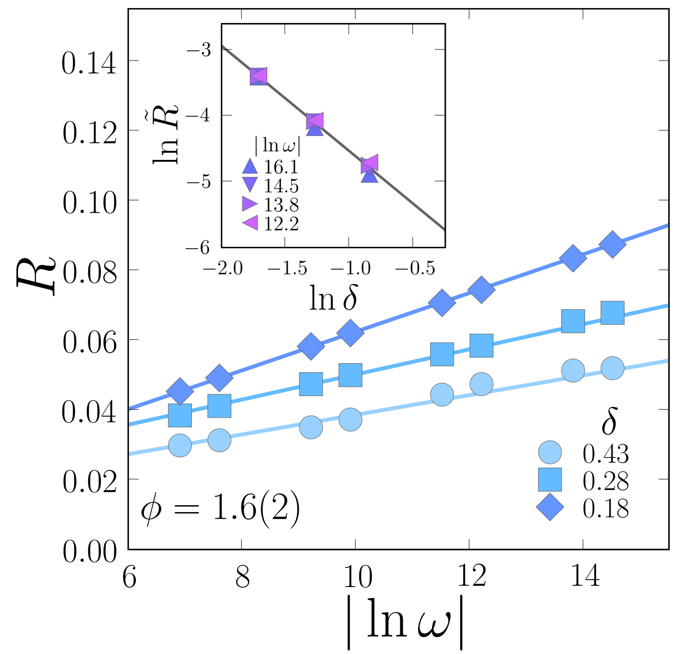
Numerical confirmation



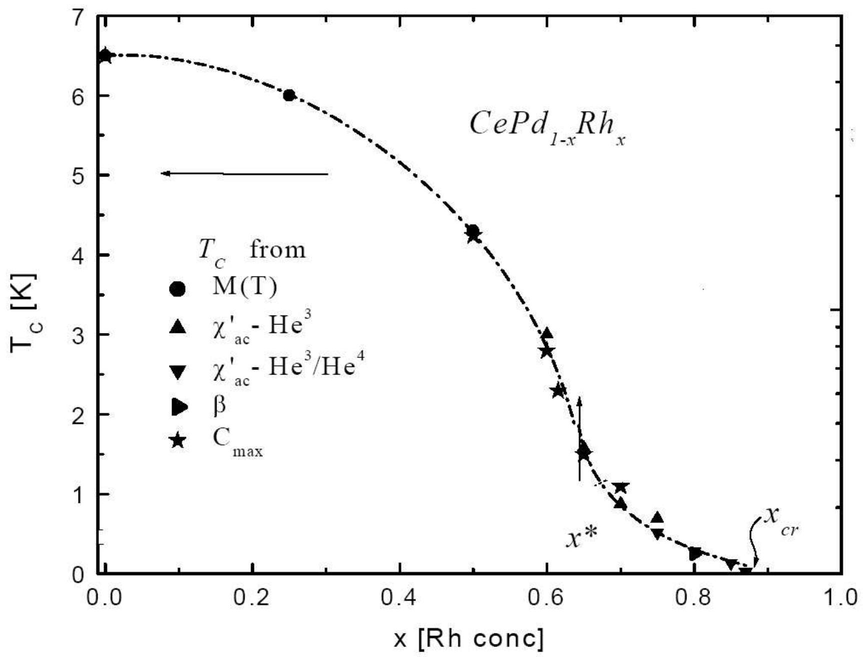
- A. Del Maestro et al. (2008) solved disordered large-N problem numerically exactly
- calculated equal time correlation function C, energy gap Ω, and ratio R of local and order parameter dynamic susceptibilities

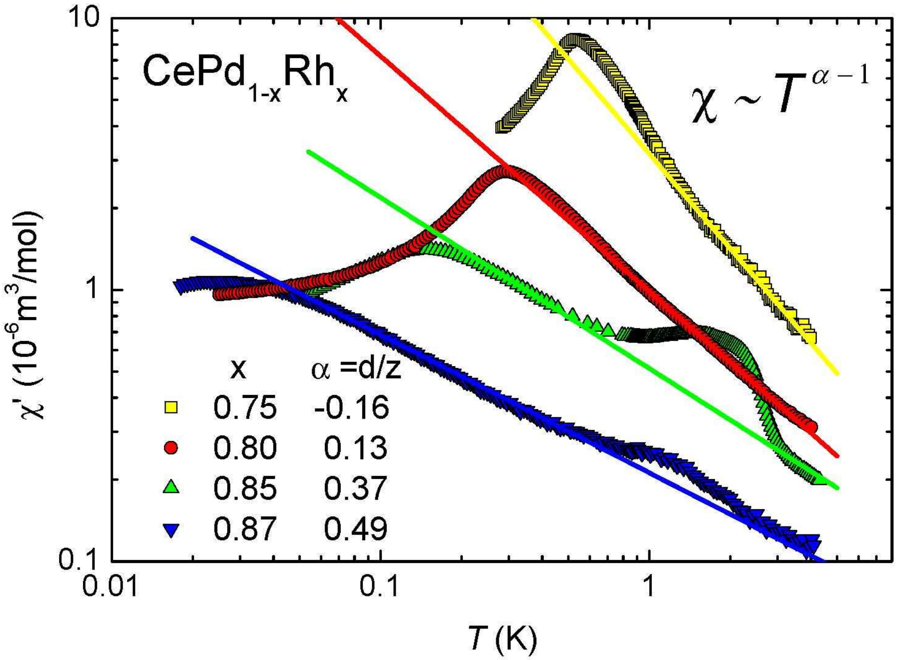
	ν	ψ	ϕ
SDRG	2	1/2	$(\sqrt{5}+1)/2$
Numerics	1.9(2)	0.53(6)	1.6(2)





Infinite-randomness physics in $CePd_{1-x}Rh_x$??





- ferromagnetic phase shows pronounced tail, evidence for glassy behavior in tail, possibly due to RKKY interactions
- above tail: nonuniversal power-laws characteristic of quantum Griffiths effects

(Sereni et al., Phys. Rev. B **75** (2007) 024432 + Westerkamp, private communication)

Classification of weakly disordered phase transitions according to importance of rare regions

T. Vojta, J. Phys. A **39**, R143–R205 (2006)

Dimensionality of rare regions	**Griffiths effects**	**Dirty critical point**	**Examples** (classical PT, QPT, non-eq. PT)
$d_{RR} < d_c^-$	weak exponential	conv. finite disorder	class. magnet with point defects dilute bilayer Heisenberg model
$d_{RR} = d_c^-$	strong power-law	infinite randomness	Ising model with linear defects random quantum Ising model disordered directed percolation (DP)
$d_{RR} > d_c^-$	RR become static	smeared transition	Ising model with planar defects itinerant quantum Ising magnet DP with extended defects

Conclusions

- We have performed a strong-disorder renormalization group study of the QPT in **disordered dissipative systems** with continuous symmetry* order parameters
- 1D: analytical solution gives **infinite-randomness** critical point in the universality class of the random transverse-field Ising model
- 2D: numerical solution displays analogous scenario, exponent values different
  3D: preliminary numerical results point in same direction
- unconventional transport properties, work in progress

For details see: Phys. Rev. Lett. **99**, 230601 (2007), Phys. Rev. B **79**, 024401 (2009),
Phys. Rev. B **80**, 041101(R) (2009)

Interplay between disorder and dissipation leads to exotic quantum critical behavior.

* There are even stronger effects for discrete symmetry that lead to a destruction of the sharp quantum phase transition by smearing, see J. A. Hoyos and T.V., Phys. Rev. Lett. **100**, 240601 (2008)