Disorder-induced rounding of certain quantum phase transitions

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- Phase transitions, disorder, and rare regions
- Rounding of quantum phase transitions in metallic systems
- Computer simulation of a model system

Phase transitions, disorder, and rare regions

Common lore:

Harris criterion, condition for homogeneous, sharp transition: $d\nu > 2$

(spatial fluctuations of local $T_c(x)$ within correlation volume must be smaller than distance from global critical point T_c)

- if clean critical point fulfills Harris criterion, it is stable against weak disorder (inhomogeneities vanish at large length scales)

even if the clean critical point is unstable (Harris criterion violated), transition is generically sharp

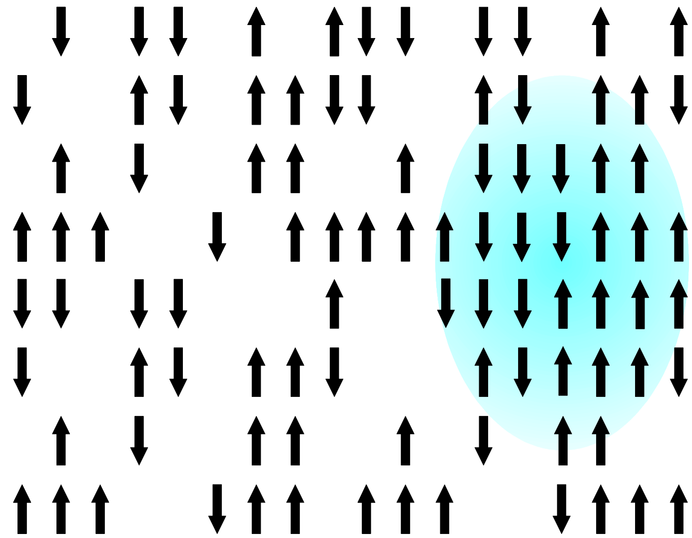
- inhomogeneities remain finite at all length scales
  $\Rightarrow$ conventional finite-disorder critical point which fulfills $d\nu > 2$
  or
- inhomogeneities diverge under coarse graining
  $\Rightarrow$ infinite-randomness critical point

Rare regions and Griffiths singularities

example: dilute ferromagnet

critical temperature T_c is reduced compared to clean value T_{c0}

for $T_c < T < T_{c0}$: no global order but local order on rare, large islands devoid of impurities



locally ordered islands have slow dynamics
$\Rightarrow$ singular free energy everywhere in the Griffiths region ($T_c < T < T_{c0}$)

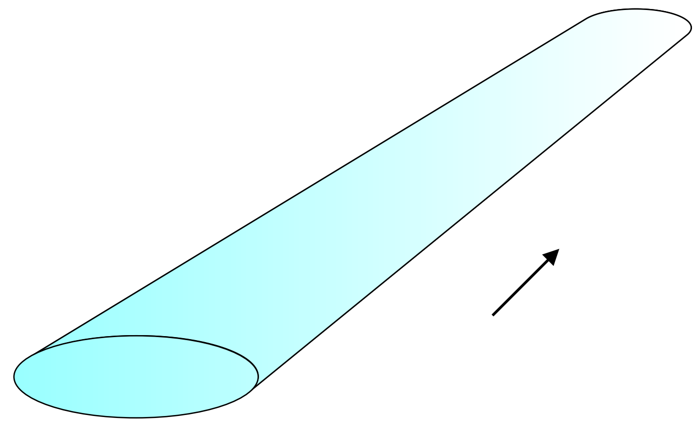
in classical systems: Griffiths singularities are generically very weak
magnetic susceptibility is finite

Disorder at quantum phase transitions

quantum phase transitions occur at zero temperature

- imaginary-time direction becomes important for critical fluctuations
- quenched disorder is totally correlated in time direction

⇒ disorder effects are enhanced



rare region at a quantum phase transition

quantum Griffiths effects

rare regions at a QPT are finite in space but infinite in imaginary time

if interaction in time direction is short-ranged, rare regions do not develop static order, but fluctuate very slowly

⇒ Griffiths singularities are enhanced

Rounding of quantum phase transitions in systems with overdamped dynamics

antiferromagnetic quantum phase transition of itinerant electrons

magnetic fluctuations are damped due to coupling to electrons

$$\Gamma(\mathbf{q}, \omega_n) = t + \mathbf{q}^2 + |\omega_n|$$

in imaginary time: long-range power-law interaction $\sim 1/(\tau - \tau')^2$

one-dimensional Ising model with $1/r^2$ interaction is known to have an ordered phase

$\Rightarrow$ in a system with overdamped dynamics and Ising symmetry, an isolated rare region can develop a static magnetization

quantum phase transition is rounded by disorder

Isolated islands – Lifshitz tail arguments

probability to find rare region of size L devoid of defects: $w \sim e^{-cL^d}$

region has transition at distance $t_c(L) < 0$ from the clean critical point
finite size scaling: $|t_c(L)| \sim L^{-\phi}$ $\quad(\phi =$ clean shift exponent)

Consequently:
probability to find a region which becomes critical at t_c:

$$w(t_c) \sim \exp(-B\ |t_c|^{-d/\phi})$$

total magnetization at coupling t is given by the sum over all rare regions having $t_c > t$:

$$m(t) \sim \exp(-B\ |t|^{-d/\phi}) \qquad (t \to 0-)$$

Computer simulation of a model system

Classical Ising model with two spatial and one time-like dimensions

quantum coupling constant $\Rightarrow$ classical temperature
original temperature $\Rightarrow$ linear size L_τ in time direction

$$H = -\frac{1}{L_\tau} \sum_{\langle \mathbf{x},\mathbf{y}\rangle,\tau,\tau'} S_{\mathbf{x},\tau} S_{\mathbf{y},\tau'} - \frac{1}{L_\tau} \sum_{\mathbf{x},\tau,\tau'} J_{\mathbf{x}} S_{\mathbf{x},\tau} S_{\mathbf{x},\tau'}$$

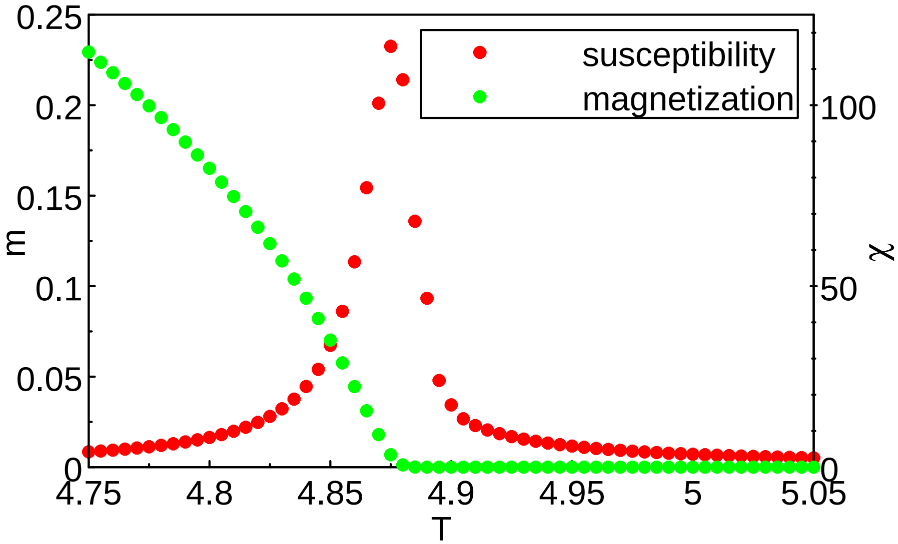
$J_{\mathbf{x}}$ – binary random variable, totally correlated in the time-like direction
$P(J) = (1-c)\,\delta(J-1) + c\,\delta(J)$

interaction in time-direction is infinite-ranged:
static magnetization on the rare regions is retained
time direction can be treated exactly, permitting large sizes

set of local mean-field equations, solved numerically be iteration

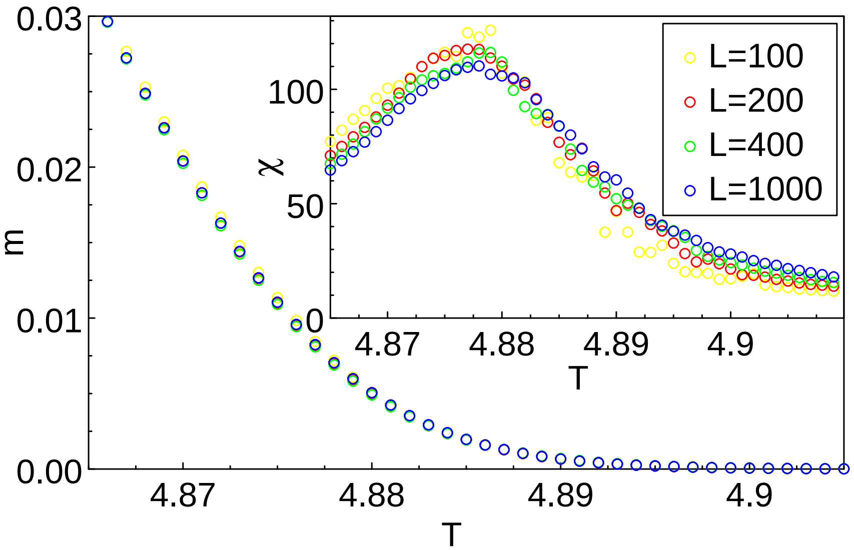
$$m_{\mathbf{x}} = \tanh \beta\, \left[J_{\mathbf{x}} m_{\mathbf{x}} + \sum_{\mathbf{y}(\mathbf{x})} m_{\mathbf{y}} + h\right]$$

Rounded transition in the infinite-range model



Magnetization + susceptibility of the infinite-range model

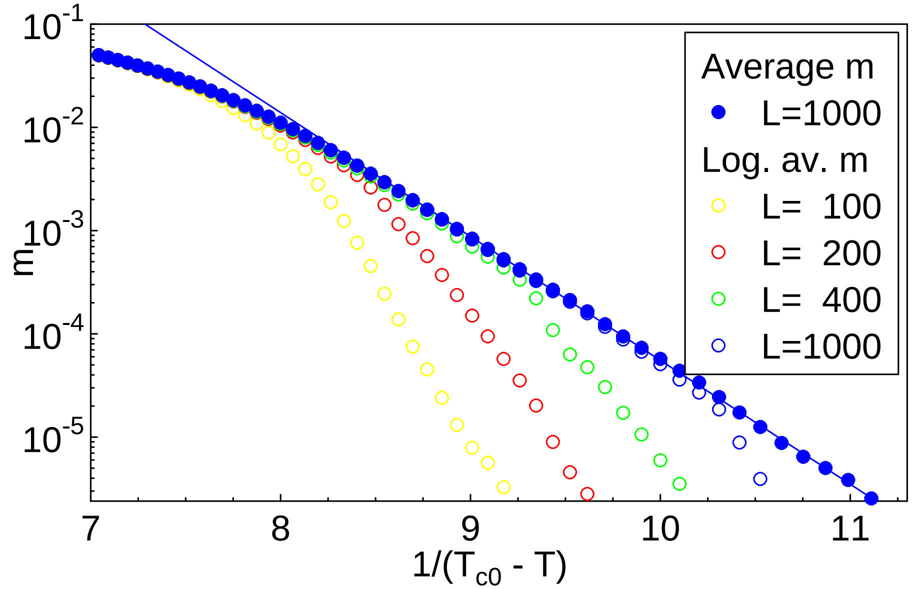
seeming transition close to $T = 4.88$



phase transition is not sharp but rounded

(m and χ are independent of L)

Magnetization in the tail



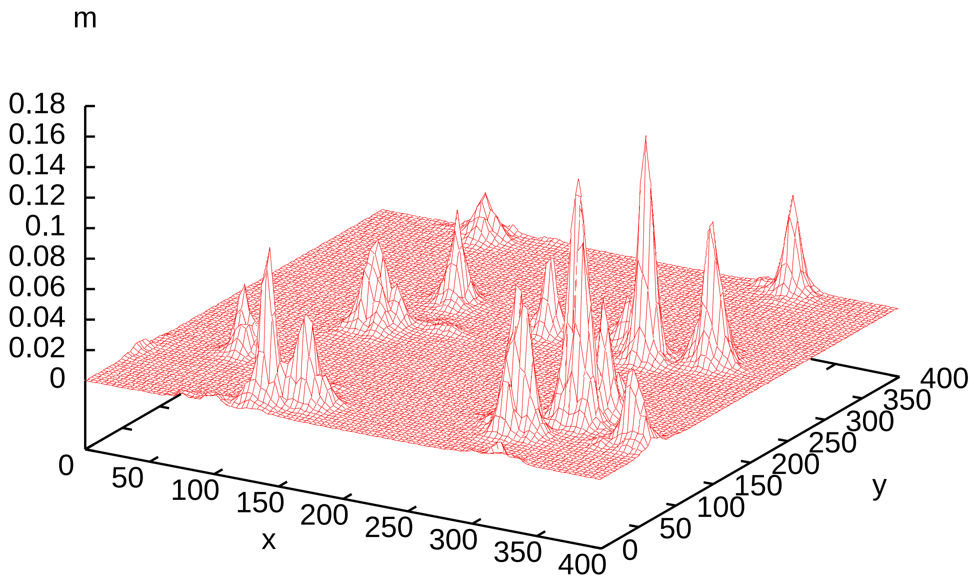
average magnetization follows the Lifshitz-tail form (solid line)

$$\log(m) \ \sim \ -1/(T_{c0} - T)$$

strong sample to sample fluctuations for

$$T_{c0} - T \ \lesssim \ 1/\log L$$

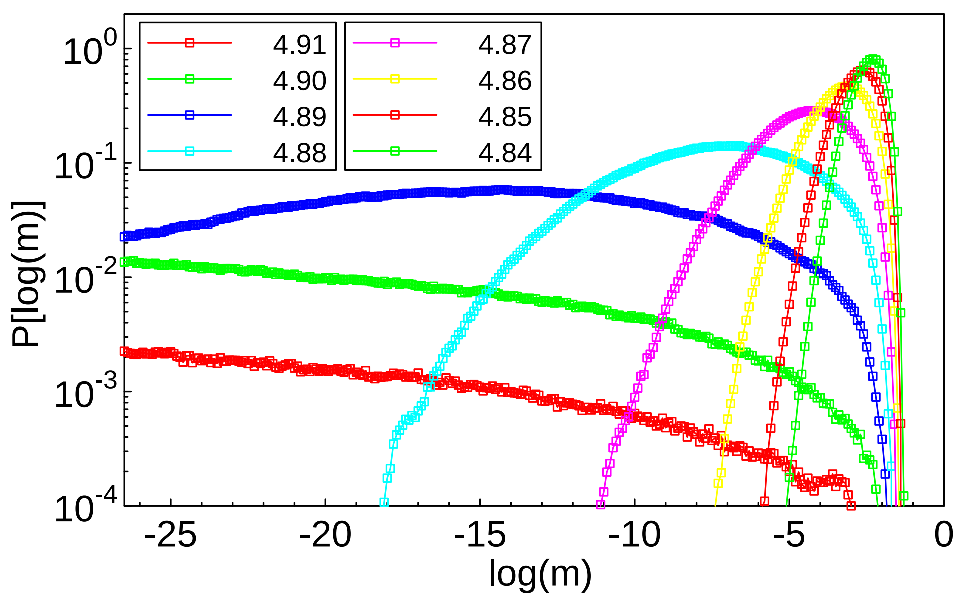
Local magnetization distribution



Local magnetization in the tail region ($T = 4.8875$)

global magnetization starts to form on isolated islands

very inhomogeneous system



Distribution of the local magnetization values

very broad, even on logarithmic scale

$$\ln(m_{typ}) \sim \langle m \rangle^{-1/2}$$

Conclusions

- quenched disorder can destroy a sharp phase transition by **rounding** if static order forms on rare spatial regions
- examples:
  - magnetic quantum phase transitions in metallic systems
  - classical Ising transitions in systems with linear defects and long-range interactions
  - classical Ising transitions with planar disorder
- exponential magnetization tail towards the non-magnetic phase, $\ln(m) \sim -|t|^{-d/\phi}$
- system is extremely **inhomogeneous**, even on a logarithmic scale
- conventional quantum Griffiths behavior does **not** exist because the rare regions are static

General mechanism for the disorder-induced rounding of phase transitions