

PROBLEMS ON HYPERSPACE RETRACTIONS

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ABSTRACT. Unsolved problems concerning hyperspace retractions for curves, dispersed in the literature, are recalled and discussed, and various interrelations between them are indicated. A particular attention is paid to retractions satisfying some additional conditions, to continuous selections on the hyperspace of subcontinua of a given curve, and to associative means, understood as a special retraction on the hyperspace of the at most two-point sets.

1. INTRODUCTION

Let a metric continuum X be given. We denote by 2^X (respectively, $C(X)$) the hyperspace of all its nonempty closed subsets (respectively, subcontinua) of X , equipped with the Hausdorff metric. Sam B. Nadler, Jr. asks in [41, (3.1), p. 193], the following question.

1.1. Problem. When is X a continuous image of 2^X or of $C(X)$?

In the same paper [41] he gives some necessary and some sufficient conditions for existence of a mapping from 2^X or from $C(X)$ onto X .

Since the hyperspace $F_1(X)$ of singletons of X is a subspace of 2^X and is homeomorphic to X , the continuum X can be considered as naturally embedded in $C(X)$. Thus, if we identify X with $F_1(X)$, we have $X \subset C(X) \subset 2^X$. So, the following is a particular case of Problem 1.1 (see [40, p. 413] and also [43, (6.2), p. 270]).

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1.2. Problem. What are necessary and sufficient conditions in order that a continuum X be a retract of 2^X or $C(X)$?

There are a number of results that are related to mappings from, onto or between some hyperspaces (which are subspaces of the hyperspace 2^X), in particular to hyperspace retractions. For example, $C(X)$ always is a continuous image of 2^X [41, Theorem 3.6, p. 194], but not necessarily a retract of 2^X , [25]. Quite a lot of these results concern local connectedness of the continuum X or of a hyperspace at some of its points (see [21], [22], [23], [27], [42] for example).

The aim of the present paper is to recall some unsolved problems related to retractions defined on hyperspaces of curves, state new ones, and indicate certain connections and interrelations between them. Given a curve X , we focus our attention on retractions from either 2^X , or $C(X)$, or $F_2(X)$ onto X , and on their special cases: selections and associative means. But we do not discuss other possible mappings between hyperspaces, in particular induced ones as well as retractions from 2^X onto other hyperspaces such as $C(X)$ or $F_n(X)$ for $n > 1$.

2. PRELIMINARIES

All considered spaces are assumed to be metric. A *continuum* means a compact connected space, and a *mapping* means a continuous function. The reader is referred to Nadler's [43] and to Illanes and Nadler's [29] books for the definitions of concepts used here and for needed information on the structure of hyperspaces.

Two special kinds of hyperspace retractions are subjects of our interest. Let X be a continuum. A retraction $r : 2^X \rightarrow X$ is said to be *associative* provided that

$$r(A \cup B) = r(\{r(A)\} \cup B) \quad \text{for every } A, B \in 2^X.$$

Let a continuum X be hereditarily unicoherent. A retraction $r : 2^X \rightarrow X$ is said to be *internal* provided that

$$r(A) \in I(A) \quad \text{for each } A \in 2^X.$$

(Here $I(A)$ means the unique continuum which is irreducible with respect to containing A .) The reader is referred to [11, Proposition

3.9 and Corollary 3.10, p. 11] for some hereditary properties of the defined concepts.

Observe that

(2.1) *each selection $\sigma : C(X) \rightarrow X$ is a retraction from $C(X)$ onto X ,*

and that

(2.2) *selectibility is a hereditary property,*

that is, if a continuum X is selectible and Y is a subcontinuum of X , then Y is selectible, too.

3. RETRACTIONS

The conditions mentioned in Problem 1.2 are known in the special case when X is locally connected. Namely the following observation is made in [40, p. 413] (see also [43, Theorem (6.4), p. 270]).

3.1. Theorem. *For a locally connected continuum X the following conditions are equivalent:*

- (3.1.1) *X is a retract of 2^X ;*
- (3.1.2) *X is a retract of $C(X)$;*
- (3.1.3) *X is an absolute retract.*

As a consequence we get a corollary.

3.2. Corollary. *A locally connected curve X is a retract of 2^X (of $C(X)$) if and only if X is a dendrite.*

For all continua, not necessarily locally connected ones, the situation is much more complicated and it does not seem that it will be clarified soon. However, there are a good deal of partial results, examples in particular, which describe various situations. Hyperspace retractions for a class of half-line compactifications are studied and many very interesting results are obtained by D. W. Curtis in [15]. Some interrelations between several conditions concerning hyperspace retractions, as well as suitable examples, are presented by S. B. Nadler, Jr. in his book [43] as well as in [29].

The following result plays an important role in our study (see [26, p. 122], and [11, Theorems 3.1 and 3.2, p. 9 and 10]).

3.3. Theorem. *If a one-dimensional continuum X is a retract of either 2^X or $C(X)$, then it is a uniformly arcwise connected dendroid.*

In [26] a complete discussion is given about all the implications concerning the existence of retractions, deformation retractions and strong deformation retractions between $F_1(X)$, $C(X)$ and 2^X ; conclusions are collected in Table I of [26, p. 130]. Results obtained in [11] and [12] erased some question marks from that table and answered some other questions from [26].

The main subject of [11] and [12] is related to Problem 1.2 of Nadler in its part concerning 2^X rather than $C(X)$. In the light of Theorem 3.1 nonlocally connected continua are under consideration. Recall that there are such continua X in all dimensions having the property that X is a deformation retract of 2^X [26, Proposition 2.10, p. 126]. Similarly like in [11] and [12] we discuss the above question under an additional assumption that $\dim X = 1$ (i.e., that the continuum X is a curve). In other words, we are interested in the following two problems, closely related to each other.

3.4. Problem. Characterize dendroids X such that X is a retract of 2^X .

3.5. Problem. Characterize dendroids X such that X is a retract of $C(X)$.

Let us recall that if a smooth dendroid X is planar, then there exists a retraction $r : 2^X \rightarrow X$, see [12, Corollary 5.4, p. 495 and Theorem 6.1, p. 496]. Further, if the smooth dendroid X can be embedded in the plane \mathbb{R}^2 so that an initial point of X (i.e., a point at which X is smooth) is accessible from the complement $\mathbb{R}^2 \setminus X$, then the retraction r exists which is both associative and internal, see [12, Theorem 6.3, p. 497]. It is not known whether the accessibility assumption is essential in this result. So we have a question (see [12, Question 6.1, p. 497]).

3.6. Question. Is it true that each planar smooth dendroid X admits an associative or internal retraction from 2^X onto X ?

The next question is related to Problems 3.4 and 3.5. It has been asked in [11, Question 3.23, p. 14].

3.7. Question. Let a dendroid X have the property of Kelley. Must then X be a retract of 2^X ? If not, determine conditions under which the answer is affirmative.

In [26, Theorem 2.9, p. 125] it is shown that every smooth fan is a deformation retract of 2^X , and in [1, Example 3.7, p. 42] a (non-smooth) plane fan X is constructed such that there is no retraction from 2^X onto X . Thus smoothness is an essential assumption in the above result. Further, the result cannot be extended to all smooth dendroids (i.e., the property of having just one ramification point is indispensable) because a (nonplanar) smooth dendroid is constructed in [11, Example 5.52, p. 25, and Remark 5.54 (a), p. 28]. On the other hand, however, it is shown in [26, Theorem 2.14, p. 129] that

(3.8) *every smooth dendroid is a deformation retract of $C(X)$.*

We will show by an example below that the converse implication to that of (3.8) is not true.

3.9. Example. *There exists a nonsmooth dendroid X which is a deformation retract of $C(X)$.*

Proof. Note that for continua X with the contractible hyperspace $C(X)$ (or, equivalently, 2^X , see e.g. [43, Theorem 16.7, p. 535]) the properties of being a retract and of being a deformation retract of $C(X)$ are equivalent. Let a dendroid X be the union of two harmonic fans H_1 and H_2 whose two points e_1 and e_2 which are accumulation points of the sets of end points are joined by an arc J having only its end points e_1 and e_2 in common with the fans (for an illustration see [13, the first picture in Fig. 2, p. 308] or [10, the left part of Fig. 11, p. 21]). Then X is selectable. Indeed, let X' be a dendroid obtained from X by shrinking the arc J to a point (equivalently, X' is homeomorphic to the one-point union of the fans H_1 and H_2 with the end points e_1 and e_2 identified). It is proved in [5, Proposition 3, p. 110] that X' is selectable. Since X' contains a homeomorphic copy of X , selectibility of X follows from the statement (2.2). Then (2.1) implies that X is a retract of $C(X)$. It is easy to observe that X is contractible, which implies contractibility of $C(X)$ according to [43, Corollary 16.8, p. 537]. Therefore X is a deformation retract of $C(X)$ by the above mentioned equivalence, but obviously X is not smooth. □

Observe that the dendroid X of Example 3.9, while nonsmooth, is pointwise smooth. Thus the following question seems to be natural.

3.10. Question. Does each pointwise smooth dendroid X admit a retraction $r : C(X) \rightarrow X$?

3.11. Remark. Note that the positive answer to Question 3.10 would imply that pointwise smooth dendroids are hereditarily contractible. Really (see [14, Remark, p. 411]), if a dendroid X is pointwise smooth, then its hyperspace $C(X)$ is contractible by [14, Theorem, p. 410]. Let $H : C(X) \times [0, 1] \rightarrow C(X)$ be a contraction. Then a mapping $h : X \times [0, 1] \rightarrow X$ defined by $h(x, t) = r(H(\{x\}, t))$ is a contraction of X ; and since the property of being a pointwise smooth dendroid is a hereditary one (see [17, (2.2), p. 198]), the conclusion follows.

On the other side, it is known that

(3.11.1) *if a continuum is hereditarily contractible, then it is a pointwise smooth dendroid.*

Indeed, each contractible continuum is contractible with respect to the circle, [32, §54, VI, Theorem 2, p. 374]; this property implies unicoherence, [32, §57, II, Theorem 2, p. 437]. Therefore each hereditarily contractible continuum is hereditarily unicoherent. Further, contractibility implies arcwise connectedness, [32, §54, VI, Theorem 1, p. 374], so the continuum is a dendroid. It is known that if a dendroid is hereditarily contractible, then it is pointwise smooth (see [16, Proposition 2, p. 170] or [17, Corollary 3.10, p. 202]).

Thereby the positive answer to Question 3.10 would give an internal characterization of hereditarily contractible continua, which is still an open problem (see [7, Problem 6.1, p. 582], [16, p. 170] and [17, (3.11), p. 202]).

In connection with Theorem 3.1 and with the comments above the following two problems can be posed.

3.12. Problems. Characterize dendroids X such that X is a deformation retract (a) of 2^X , (b) of $C(X)$.

4. SELECTIONS

It was shown in [33] that a continuum X admits a selection on the hyperspace 2^X if and only if X is an arc. For the hyperspace $C(X)$ no such characterization does exist. One of the important steps forward was made by Nadler and Ward who have shown in [44, Lemma 3, p. 370] that if a continuum X admits a selection on $C(X)$ (i.e., if it is selectable), then it is a dendroid. Since such a selection is a retraction, (2.1), it follows from Theorem 3.3 that the dendroid X is uniformly arcwise connected. Thus the main problem in this topic is the following.

4.1. Problem. Give an internal (structural) characterization of selectable dendroids.

According to (2.1) the problem of finding necessary and sufficient conditions concerning the structure of the continuum X for the existence of a selection for the hyperspace $C(X)$ is related to much wider Problems 1.2 and 3.5. For a discussion of various necessary or sufficient conditions the reader is referred to [44], [37], [38] and expository papers [6] and [9], where a number of references are given.

The following question is related to Question 3.10.

4.2. Question. Does each pointwise smooth dendroid X admit a selection $\sigma : C(X) \rightarrow X$?

Note that the opposite implication does not hold: the (auxiliary) dendroid X' described in the proof of Example 3.9 is selectable, see [5, Proposition 3, p. 110], while obviously it is not pointwise smooth.

Observe also that if Question 4.2 would be answered in the affirmative, then also Question 3.10 would have the positive answer by (2.1), and therefore we would have an internal characterization of hereditarily contractible dendroids according to Remark 3.11. Such a characterization is known for fans. The following result is shown in [46, Theorem 2, p. 1043].

(4.3) *A continuum X is a smooth dendroid if and only if there exists a rigid selection for the hyperspace $C(X)$ of its subcontinua.*

Now the mentioned characterization runs as follows (see [9, (15), p. 95]).

4.4. Theorem. *For each fan X the following conditions are equivalent:*

- (4.4.1) X is smooth;
- (4.4.2) X is pointwise smooth;
- (4.4.3) X is hereditarily contractible;
- (4.4.4) there exists a rigid selection for $C(X)$.

Studying connections between various properties of dendroids (as e.g. contractibility, selectibility, existence of some special retractions from a hyperspace onto the continuum, and others) a very interesting dendroid (that accumulates many properties in one example) has been constructed in [38, Example, p. 321] and in [28, Section 4, p. 70]. In particular it gives a negative answer to a question asked by Nadler in [43, (5.11), p. 259] if every contractible dendroid is selectible. Its construction and picture is repeated, and properties are collected, in [11, p. 28-32, in particular Theorem 5.78, p. 31]. However, the example is rather complicated in the sense that it is nonplanar and it has countably many ramification points. Thus it would be interesting to know if there is a simpler example having the same properties. In particular, we have the following questions (see [9, Questions 11 and 12, p. 94]).

4.5. Questions. Does there exist a contractible and nonselectible dendroid X with the property that there exists a retraction from $C(X)$ onto X and which either (a) is planar, or (b) has finitely many ramification points?

Nadler's question mentioned above remains open if we require that the dendroid in matter is a fan.

4.6. Question. Does there exist a contractible and nonselectible fan?

This question has the negative answer if we demand hereditary contractibility in place of contractibility, as it can be seen from Theorem 4.4.

5. MEANS

Recall that a *mean* on a space X is a mapping $\mu : X \times X \rightarrow X$ such that $\mu(x, y) = \mu(y, x)$ and $\mu(x, x) = x$ for every $x, y \in X$. It is evident that the existence of a mean on a continuum X is equivalent to the existence of a retraction $r_2 : F_2(X) \rightarrow X$, because we can define $r_2(\{x, y\}) = \mu(x, y)$ (or vice versa), see [43, Theorem 6.17, p. 285]. A mean μ is said to be *associative* provided that $\mu(x, \mu(y, z)) = \mu(\mu(x, y), z)$ for every $x, y, z \in X$. A mean μ defined on a dendroid X is said to be *internal* provided that $\mu(x, y) \in xy$ for every $x, y \in X$, where xy denotes the (unique) arc joining x and Y in X . The reader is referred to [43, p. 285], [1], [30], [8], or [11, Chapter 5, p. 18] for more information on means. In particular, the papers [1] and [30] contain criteria for the nonexistence of means on given continua.

The fundamental problem concerning means is the following (see [43, Question 6.17.1, p. 285] and [11, Problems 5.28, p. 21]).

5.1. Problem. Characterize topological spaces (in particular metric continua) that admit a mean (or an associative mean).

Recall that a one-dimensional locally connected continuum admits a mean if and only if it is a dendrite, see [45, p. 65] and compare [11, Proposition 5.30, p. 22]. However, the following questions are open (see [11, Question 5.38, p. 23]).

5.2. Questions. Let a continuum X be locally connected. (a) Does the existence of a mean on X imply the existence an associative mean on X ? (b) Does the existence an associative mean on X imply that X is an absolute retract?

The following results are known, [11, Proposition 5.16 and Theorem 5.21, p. 20, and Corollary 5.41, p. 24].

5.3. Theorem. (a) *Let X be a dendroid. If there exists a retraction $r : 2^X \rightarrow X$, then the mapping $\mu : X \times X \rightarrow X$ defined by $\mu(x, y) = r(\{x, y\})$ is a mean on X . Moreover, if r is internal (associative), then so is μ .*

(b) *Let a continuum X be either one-dimensional or hereditarily unicoherent. If X admits an associative mean, then X is a smooth dendroid.*

(c) *Every smooth fan admits an associative and internal mean.*

In the light of Theorem 5.3 (a) one can say that for a dendroid X the existence of a retraction $r : 2^X \rightarrow X$ is a stronger condition than the existence of a mean $\mu : X \times X \rightarrow X$. But we do not know if the inverse implication holds. Precisely, the following question is interesting (see [11, Question 5.44, p. 24]).

5.4. Question. Does there exist a dendroid X which admits a mean $\mu : X \times X \rightarrow X$ and for which there is no retraction $r : 2^X \rightarrow X$?

The existence of a retraction $r : 2^X \rightarrow X$, where X is a dendroid, implies its uniform connectedness, see Theorem 3.3. The next questions are related to this result and to Questions 3.7, [11, Questions 5.48 and 5.49, p. 25].

5.5. Questions. (a) Let a dendroid X admit a mean. Must then X be uniformly arcwise connected?

(b) Let a dendroid X have the property of Kelley. Does there exist a mean on X ?

The inverse implication to the one in Theorem 5.3 (b) is not true because a smooth dendroid does exist which admits no mean, [11, Example 5.52, p. 25]. The dendroid is not planar, and this property is essential: each planar smooth dendroid admits a mean, [12, Theorem 6.6, p. 497].

In the light of above quoted results and questions the following problems can be posed that supply Problem 5.1.

5.6. Problems. Characterize (a) dendroids, (b) smooth dendroids, (c) planar dendroids, which admit a mean.

REFERENCES

1. M. Bell and S. Watson, *Not all dendroids have a mean*, *Houston J. Math.* **22** (1996), 39–50.
2. D. P. Bellamy, *The cone over the Cantor set—continuous maps from both directions*, *Proc. Topology Conference* (J. W. Rogers, Jr., ed.), Emory University, Atlanta, Ga., 1970, pp. 8–25.
3. J. J. Charatonik, *Two invariants under continuity and the incomparability of fans*, *Fund. Math.* **53** (1964), 184–204.
4. J. J. Charatonik, *Problems and remarks on contractibility of curves*, *General Topology and its Relations to Modern Analysis and Algebra IV*, *Proceedings of the Fourth Prague Topological Symposium 1976, Part B Contributed Papers*, Society of Czechoslovak Mathematicians and Physicists, Prague, 1977, pp. 72–76.

5. J. J. Charatonik, *Contractibility and continuous selections*, Fund. Math. **108** (1980), 109–118.
6. J. J. Charatonik, *Conditions related to selectibility*, Math. Balkanica (N. S.) **5** (1991), 359–372.
7. J. J. Charatonik, *Contractibility of curves*, Matematiche (Catania) **46** (1991), 559–592.
8. J. J. Charatonik, *Some problems concerning means on topological spaces*, Topology, Measures and Fractals, Mathematical Research vol. 66, Akademie Verlag, Berlin, 1992, pp. 166–177.
9. J. J. Charatonik, *On continuous selections for the hyperspace of subcontinua*, Topology (Pécs 1989) (A. Császár, ed.), Colloq. Math. Soc. János Bolyai **55**, North-Holland, Amsterdam and New York, 1993, pp. 91–100.
10. J. J. Charatonik, *On acyclic curves. A survey of results and problems*, Bol. Soc. Mat. Mexicana (3) **1** (1995), 1–39.
11. J. J. Charatonik, W. J. Charatonik, K. Omiljanowski and J. R. Prajs, *Hyperspace retractions for curves*, Dissertationes Math. (Rozprawy Mat.) **370** (1997), 1–34.
12. J. J. Charatonik, W. J. Charatonik, K. Omiljanowski and J. R. Prajs, *On plane arc-smooth structures*, Houston J. Math. **25** (1999), 473–499.
13. J. J. Charatonik and C. Eberhart, *On smooth dendroids*, Fund. Math. **67** (1970), 297–322.
14. W. J. Charatonik, *Pointwise smooth dendroids have contractible hyperspaces*, Bull. Polish Acad. Sci. Math. **33** (1985), 409–412.
15. D. W. Curtis, *A hyperspace retraction theorem for a class of half-line compactifications*, Topology Proc. **11** (1986), 29–64.
16. S. T. Czuba, *A concept of pointwise smooth dendroids*, Russian Math. Surveys **34:6** (1979), 169–171, Russian original in Uspekhi Mat. Nauk **34:6** (1979), 215–217.
17. S. T. Czuba, *On pointwise smooth dendroids*, Fund. Math. **114** (1981), 197–207.
18. S. T. Czuba, *Some other characterizations of pointwise smooth dendroids*, Comment. Math. Prace Mat. **24** (1984), 195–200.
19. S. T. Czuba, *On dendroids for which smoothness, pointwise smoothness and hereditary contractibility are equivalent*, Comment. Math. Prace Mat. **25** (1985), 27–30.
20. S. T. Czuba, *On dendroids with Kelley's property*, Proc. Amer. Math. Soc. **102** (1988), 728–731.
21. R. B. Fraser, *A new characterization of Peano continua*, Comment. Math. Prace Mat. **16** (1972), 247–248.
22. J. T. Goodykoontz, Jr., *Connectedness in kleinen and local connectedness in 2^X and $C(X)$* , Pacific J. Math. **53** (1974), 387–397.
23. J. T. Goodykoontz, Jr., *More on connectedness in kleinen and local connectedness in $C(X)$* , Proc. Amer. Math. Soc. **65** (1977), 357–364.
24. J. T. Goodykoontz, Jr., *Some functions on hyperspaces of hereditarily unicoherent continua*, Fund. Math. **95** (1977), 1–10.

25. J. T. Goodykoontz, Jr., *$C(X)$ is not necessarily a retract of 2^X* , Proc. Amer. Math. Soc. **67** (1977), 177–178.
26. J. T. Goodykoontz, Jr., *Some retractions and deformation retractions on 2^X and $C(X)$* , Topology Appl. **21** (1985), 121–133.
27. J. T. Goodykoontz, Jr. and S. B. Nadler, Jr., *Whitney levels in hyperspaces of certain Peano continua*, Trans. Amer. Math. Soc. **274** (1982), 671–694.
28. A. Illanes, *Two examples concerning hyperspace retraction*, Topology Appl. **29** (1988), 67–72.
29. A. Illanes and S. B. Nadler, Jr., *Hyperspaces: Fundamentals and Recent Advances*, M. Dekker, New York and Basel, 1999.
30. K. Kawamura and E. D. Tymchatyn, *Continua which admit no mean*, Colloq. Math. **71** (1996), 97–105.
31. J. L. Kelley, *Hyperspaces of a continuum*, Trans. Amer. Math. Soc. **52** (1942), 22–36.
32. K. Kuratowski, *Topology*, vol. 2, Academic Press and PWN, New York, London and Warszawa, 1968.
33. K. Kuratowski, S. B. Nadler, Jr. and G. S. Young, *Continuous selections on locally compact separable metric spaces*, Bull. Acad. Polon. Sci. Ser. Sci. Math. Astronom. Phys. **18** (1970), 5–11.
34. W. Kuperberg, *Uniformly pathwise connected continua*, Studies in Topology, Proc. Conf. Univ. North Carolina, Charlotte, NC, 1974, Academic Press, New York, 1975, pp. 315–324.
35. A. Lelek, *On weakly chainable continua*, Fund. Math. **51** (1962), 271–282.
36. L. Lum, *Weakly smooth dendroids*, Fund. Math. **83** (1974), 111–120.
37. T. Maćkowiak, *Continuous selections for $C(X)$* , Bull. Acad. Polon. Sci. Ser. Sci. Math. Astronom. Phys. **21** (1978), 547–551.
38. T. Maćkowiak, *Contractible and nonselectible dendroids*, Bull. Polish Acad. Sci. Math. **33** (1985), 321–324.
39. H. C. Miller, *On unicoherent continua*, Trans. Amer. Math. Soc. **69** (1950), 179–194.
40. S. B. Nadler, Jr., *Inverse limits and multicoherence*, Bull. Amer. Math. Soc. **76** (1970), 411–414.
41. S. B. Nadler, Jr., *Some problems concerning hyperspaces*, Proc. Topology Conference (V. P. I. and S. U., 1973, R. F. Dickman, Jr. and P. Fletcher, eds.), Lecture Notes in Math., vol. 375, Springer Verlag, New York, 1974, pp. 190–197.
42. S. B. Nadler, Jr., *A characterization of locally connected continua by hyperspace retractions*, Proc. Amer. Math. Soc. **67** (1977), 167–176.
43. S. B. Nadler, Jr., *Hyperspaces of Sets*, M. Dekker, 1978.
44. S. B. Nadler, Jr. and L. E. Ward, Jr., *Concerning continuous selections*, Proc. Amer. Math. Soc. **25** (1970), 369–374.
45. K. Sigmon, *A note on means in Peano continua*, Aequationes Math. **1** (1968), 85–86.
46. L. E. Ward, Jr., *Rigid selections and smooth dendroids*, Bull. Acad. Polon. Sci. Ser. Sci. Math. Astronom. Phys. **19** (1971), 1041–1044.

47. M. Wojdysławski, *Sur la contractilité des hyperspaces des continus localement connexes*, Fund. Math. **30** (1938), 247–252.
48. M. Wojdysławski, *Rétractes absolus et hyperspaces des continus*, Fund. Math. **32** (1939), 184–192.

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