PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 118, Number 4, August 1993

CONTINUA AS POSITIVE WHITNEY LEVELS

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(Communicated by James E. West)

ABSTRACT. It is shown that every continuum is a positive Whitney level of some continuum.

Krasinkiewicz and Nadler have asked (independently) if every continuum is a positive Whitney level of some continuum. The question has never been published, but it is known for people working in continuum theory. Here we present a very simple proof of it, which is in fact a compilation of known results.

For a given continuum X we denote by $C(X)$ the hyperspace of all subcontinua of X with the Hausdorff metric. A *Whitney map* is a map $\omega\colon C(X) \to [0, \infty)$ that satisfies the following two conditions:

(1) $\omega(\{x\}) = 0$ for all $x \in X$, and
(2) if $A, B \in C(X)$ satisfy $A \subseteq B$ and $A \neq B$ then $\omega(A) < \omega(B)$.

We will also use the notion of an atomic continuum. A subcontinuum A of a continuum X is called *atomic* if for any subcontinuum B of X we have $A \subseteq B$ or $B \subseteq A$. In newer papers atomic continua are also called *terminal*.

Theorem. *Let X be any continuum. Then there exist a continuum M, a Whitney map $\omega\colon C(M) \to [0, \infty)$, and a number $t \in (0, \omega(M))$ such that X is homeomorphic to $\omega^{-1}(t)$.*

Proof. By [1, Theorem, p. 507] there exist a continuum M and a monotone open map $f\colon M \to X$ such that all point inverses $f^{-1}(x)$ for $x \in X$ are nondegenerate atomic subcontinua of M. Let

$$\mathcal{M} = \{\{m\}\colon m \in M\} \cup \{f^{-1}(x) : x \in X\} \cup \{M\}.$$

Thus $\mathcal{M}$ is a compact subset of $C(M)$. Define $w\colon \mathcal{M} \to [0, \infty)$ by $w(\{m\}) = 0$, for $m \in M$, $w(f^{-1}(x)) = 1$, and $w(M) = 2$. Then w is continuous, and by [3, Corollary 3.4, p. 468] it can be extended to a Whitney map $\omega\colon C(M) \to [0, \infty)$. Because all point inverses $f^{-1}(x)$ for $x \in X$ are atomic continua, we have $\omega^{-1}(1) = \{f^{-1}(x) : x \in X\}$ and therefore $\omega^{-1}(1)$ is homeomorphic to X. The proof is complete.

Received by the editors March 27, 1992.
1991 *Mathematics Subject Classification.* Primary 54B20.
Key words and phrases. Atomic decomposition, continuum, hyperspace, Whitney level, Whitney map.

ACKNOWLEDGMENT

The author would like to thank Paweł Krupski for help in finding the appropriate reference.

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