BULLETIN OF THE POLISH
ACADEMY OF SCIENCES
MATHEMATICS
Vol. 50, No. 1, 2002

GENERAL TOPOLOGY

On Open Mappings of Compactifications of the Ray onto an Arc

by

Janusz J. CHARATONIK, Włodzimierz J. CHARATONIK
and Pavel PYRIH

Presented by Czesław BESSAGA on March 7, 2001

Summary. The existence of an open mapping from a compactification of a ray onto the closed unit interval $[0,1]$ is investigated. The main theorem says that such an open mapping does exist if and only if there is a confluent mapping of the remainder onto $[0,1]$.

The following problem is posed in [5, p. 41].

PROBLEM 1. *What continua can be openly mapped onto an arc?*

Open mappings from a locally connected continuum onto an arc have been investigated by a number of authors, in particular by G. T. Whyburn in his book [17]. Further studies on the problem led to some characterizations of locally connected continua which admit an open mapping onto an arc, see [5]. For all continua, not necessarily locally connected ones, the situation is much more complicated and seems to be far from any complete solutions. In these circumstances even partial results, theorems as well as examples, have some value, and they could be helpful either in finding a full solution of Problem 1 or (at least) in describing other classes of spaces (in particular of continua) which admit an open mapping onto an arc.

One of the simplest nonlocally connected continua are compactifications X of the ray C having a continuum B as the remainder, i.e. of the form $X \doteq B \cup C$ with C being a one-to-one image of the real half-line. They are

---

2000 MS Classification: 54E40, 54F15, 54F50.

Key words: arc, continuum, confluent, mapping, open, ray, unit interval.

The third named author was supported by CEZ J13/98113200007 and GAUK 165/99 Charles University.

natural generalizations of the $\sin(1/x)$-curve. Mappings of some of them were studied earlier, see e.g. [15], where all confluent images of the $\sin(1/x)$-curve are determined. Recently other related results have been obtained. In [7] two large families, **L** and **H**, of continua are exhibited of the form $A \cup B \cup C$, where B and C are as above and A is a continuum such that $A \cap B \neq \emptyset$ having the properties that each open mapping defined on a member of **L** is light, and each open mapping defined on a member of **H** is a homeomorphism. Further, it is shown in [7, Remark 18, p. 242] that $\sin(1/x)$-curve is not in the class **L**. The property of being in the class **L** is not preserved under atomic mappings, [8, Example 2.10, p. 162]. Thus, in connection with Problem 1, a natural question can be asked what continua of the form either $A \cup B \cup C$ or $B \cup C$, with the meaning of A, B and C as above, admit an open mapping onto an arc, and which of these continua admit an open nonlight mapping. This is just the subject of the present paper. The research can be understood as a prolongation and completion of other studies in the area, as e.g. [6], [14] or [4].

All spaces considered in the paper are assumed to be metric and all mappings are continuous. A *continuum* means a compact connected space. Given a space X and its subset S, we denote by $\operatorname{cl} S$ the closure of S and by $\operatorname{int} S$ its interior in X. As usual $\mathbb{N}$ denotes the set of positive integers, and $\mathbb{R}$ stands for the space of real numbers.

A subset S of a space X is said to be *arcwise connected* provided that for every two points p and q of S there exists in X an arc A with end points p and q such that $A \subset S$.

A continuum is said to be *irreducible* provided that it contains two points a and b such that no proper subcontinuum of it contains these points. Then we say that the continuum is irreducible *between a and b*, and we denote the continuum by $I(a, b)$. It is known that any continuum containing some two points contains a subcontinuum that is irreducible between them, see [12, Paragraph 48, I, Theorem 1, p. 192].

Recall that a subcontinuum B of a continuum X is said to be *terminal* (in X) provided that for each subcontinuum A of X intersecting B we have either $A \subset B$ or $B \subset A$. Note that, in particular, the whole continuum X is terminal in X. This concept, used e.g. by Maćkowiak and Tymchatyn in [13, p. 17], should not be confused with other ones, known under the same name (see Bennett and Fugate [2, Definition 1.1, p. 7]; Fugate [10, p. 461]; Gordh [11, p. 458]; compare Nadler's book [16, (1.54), p. 107] and a discussion in [2, p. 35 and 39]).

A surjective mapping $f : X \to Y$ between topological spaces is said to be:

- *open* provided that the images of open sets under f are open;
- *monotone* provided that for each point $y \in Y$ the set $f^{-1}(y)$ is connected;

– *light* provided that for each point $y \in Y$ the set $f^{-1}(y)$ has one-point components (note that if the point-inverses are compact, this condition is equivalent to the property that they are zero-dimensional);
– *confluent* provided that for each subcontinuum Q of Y each component of $f^{-1}(Q)$ is mapped onto Q under f.

Let the symbol **NO** (the symbol **NC**) stand for the class of continua that admit no open (no confluent, respectively) mapping onto an arc. For example, since hereditary indecomposability is an invariant property under confluent mappings (see [16, (1.207.5), p. 198])

(2) *each hereditarily indecomposable continuum is in* **NC**.

To prove the next result we need the following two propositions.

PROPOSITION 3. *The concept of a terminal continuum is invariant under confluent mappings, i.e. if a mapping $f : X \to Y$ between continua is confluent, and if B is a terminal subcontinuum of X, then $f(B)$ is a terminal subcontinuum of Y.*

Proof. Let Q be a subcontinuum of Y such that $Q \cap f(B) \neq \emptyset$. Then $B \cap f^{-1}(Q) \neq \emptyset$, so there exists a component C of $f^{-1}(Q)$ such that $C \cap B \neq \emptyset$. Since B is terminal in X, either $B \subset C$, which implies $f(B) \subset f(C) = Q$, or $C \subset B$, which implies $Q = f(C) \subset f(B)$, by the confluence of f. The proof is complete.

PROPOSITION 4. *If a mapping $f : X \to Y$ between continua is confluent, and if B is a terminal subcontinuum of X such that $f(B) = Y$, then the restriction $f|B : B \to Y$ also is confluent.*

Proof. Let Q be a subcontinuum of Y, and let C be a component of $(f|B)^{-1}(Q) = B \cap f^{-1}(Q)$. Denote by D the component of $f^{-1}(Q)$ that contains C. Since B is terminal in X, then either $B \subset D$, or $D \subset B$. In the first case we have $Y = f(B) \subset f(D) = Q$, whence $Q = Y$ and consequently $(f|B)^{-1}(Q) = B = C$, and $(f|B)(C) = Q$ as needed. In the second case, i.e. if $D \subset B$, we see that $D = C$, which implies $(f|B)(C) = f(C) = f(D) = Q$ again. Thus $f|B$ is confluent.

If C is a dense subspace of a compact space Z, then Z is called a *compactification* of C, and $Z \setminus C$ is called the *remainder* of C in Z (see e.g. [1, p. 34]).

PROPOSITION 5. *For each compactification of the ray C with a continuum $B \in$ **NC** as the remainder, and for each confluent mapping $f : X = B \cup C \to [0, 1]$ the image $f(B)$ is a singleton.*

Proof. Observe that B is a proper terminal subcontinuum of X, and that the unit segment $[0,1]$ has three terminal subcontinua: the two singletons being the end points, and the whole $[0,1]$. If $f(B)$ would not be a singleton, then we would have $f(B) = [0,1]$ by Proposition 3, and therefore the restriction $f|B : B \to [0,1]$ would be confluent by Proposition 4, contrary to the assumption that $B \in \mathbf{NC}$. This finishes the proof.

THEOREM 6. *Let a continuum B be in the class* **NC**. *Then for each compactification $X = B \cup C$ of the ray C with B as the remainder, there is no open mapping from X onto an arc, i.e. $X \in$* **NO**.

Proof. Suppose on the contrary that there is a nonconstant open mapping $f : X \to [0,1]$. Since (for compact spaces) each open mapping is confluent (see [17, Theorem (7.5), p. 148]; compare [3, VI, p. 214]), the image $f(B)$ is a singleton by Proposition 4. Therefore the mapping f can be factored, i.e. expressed as the composition $f = f_2 \circ f_1$ of two mappings $f_1 : X \to A$ and $f_2 : A \to [0,1]$ such that $f_1 : X \to X/B = A$ is a monotone mapping that shrinks the continuum B to a point (and is a homeomorphism on $X \setminus B = C$). Thus the range space A is an arc. Since the continuum X is irreducible from each point of B to the end of C and since monotone mappings preserve points of irreducibility, see [12, Paragraph 48, I, Theorem 3, p. 192], the point $f_1(B)$ is an end point of the arc A. Without loss of generality we may assume that $A = [0,1]$ and that $f_1(B) = \{0\}$.

Further, since the composition $f = f_2 \circ f_1$ is open, and since the class of open mappings of compact spaces has the composition factor property (i.e. if the composition f is open, then the second mapping, f_2, is also open, see [17, (3.1), p. 140]), we conclude that $f_2 : A = [0,1] \to [0,1]$ is an open mapping. Using Whyburn's characterization of an open mapping defined on an arc, [17, (1.3), p. 184] we see that there is $\varepsilon > 0$ such that $f_2|[0,\varepsilon] : [0,\varepsilon] \to f_2([0,\varepsilon]) \subset [0,1]$ is a homeomorphism. Therefore we infer that if $x \in B$ and if U is an open neighbourhood of x in X that is small enough (i.e. is such that an infinite sequence of subarcs of the ray C lie outside of U and some limit points of this sequence are in B), then its image $f(U)$ is not an open subset of $[0,1]$, a contradiction. The proof is complete.

By statement (2) we get the following corollary.

COROLLARY 7. *For each compactification $X = B \cup C$ of the ray C with a hereditarily indecomposable continuum B as the remainder we have $X \in$* **NO**.

Remark 8. As an example of a hereditarily decomposable continuum in the class **NC** (thus in **NO**) one can take the Lelek fan F_L, which has the property that each nondegenerate confluent image of F_L is homeomorphic to F_L, see [9, Proposition 2, p. 32], and thereby it has no confluent (hence no open) mapping onto an arc.

Remark 8 and Theorem 6 imply the following corollary.

COROLLARY 9. *For each compactification* $X = B \cup C$ *of the ray* $Ç$ *with the Lelek fan* $F_L = B$ *as the remainder we have* $X \in \mathbf{NO}$.

To show the converse to Theorem 6 we need an auxiliary result. The result is related to the following theorem of G. T. Whyburn (see [17, Theorem (2.1), p. 186]).

THEOREM 10 (Whyburn). *If a mapping* $f : X \to Y$ *between compact spaces* X *and* Y *is light open, then for each arc* $pq \subset Y$ *and for each point* $a \in f^{-1}(p)$ *there is an arc* $ab \subset X$ *such that the restriction* $f|ab : ab \to pq$ *is a homeomorphism.*

To prove the result we recall some notation. Given a compact space X with a metric d, a subset $A \subset X$ and a positive number ε, let $N_X(A, \varepsilon)$ stand for an ε-neighbourhood of A in X with the radius ε, i.e. $N_X(A, \varepsilon) = \{x \in X : d(x, A) < \varepsilon\}$. To simplify further formulations, for a given (ordered) arc L we denote by $\widetilde{L}$ the same arc ordered invertedly.

PROPOSITION 11. *Let* $f : A \to [0,1]$ *be a light open mapping of a continuum* A *with a metric* d *onto the closed unit interval* $[0,1]$, *let* $a \in f^{-1}(0)$ *and* $p \in A$. *Then for each* $\varepsilon > 0$ *there exist a positive integer* n *and a finite sequence of* n *arcs* $A_i = a_i b_i \subset A$, *where* $i \in \{1, \dots, n\}$, *satisfying the following conditions:*

(11.1) *the restrictions* $f|A_i : A_i \to [0,1]$ *are embeddings for each* $i \in \{1, \dots, n\}$;

(11.2) $a_1 = a$ *and, for each* $i \in \{1, \dots, n-1\}$, *we have* $d(b_i, a_{i+1}) < \varepsilon$ *and* $f(b_i) = f(a_{i+1})$;

(11.3) *for each* $i \in \{1, \dots, n-1\}$ *we have either* $f(A_i) \cap f(A_{i+1}) = \{f(b_i)\}$ *or* $f(b_i) \in \{0,1\}$, *and* $f(b_n) \in \{0,1\}$;

(11.4) $p \in A_k$ *for some* $k \in \{1, \dots, n\}$ *and, if* $f(p) \notin \{0,1\}$, *then* $p \in A_k \setminus \{a_k, b_k\}$;

(11.5) $b_n = a$;

(11.6) $\operatorname{diam} A_i < \varepsilon$ for each $i \in \{1, \dots, n\}$.

Proof. For a given $\varepsilon > 0$ denote by $\mathcal{A}$ the family of all finite sequences of arcs satisfying conditions (11.1)–(11.3), and let U be the union of all arcs being elements of members of $\mathcal{A}$. We will prove that $U = A$ by showing that U is both open and closed subset of A.

We will say that a *sequence of arcs passes through a point* provided that the point belongs to the union of the elements of the sequence.

First we show that the set U is open. If $x \in U$, then there is a sequence $\{A_1, \dots, A_m\}$ of arcs satisfying conditions (11.1)–(11.3), and $x \in A_k$ for some $k \in \{1, \dots, m\}$. Since the mapping $(f|A_j)^{-1}$ is uniformly continuous for each $j \in \{1, \dots, m\}$, the set

$$V_k = \{y \in A : \text{ there is a point } z \in A_k \text{ such that } f(z) = f(y) \text{ and } d(z, y) < \varepsilon\}$$

is a neighbourhood of x if $x \in A_k \setminus \{a_k, b_k\}$. Moreover, $V_{k-1} \cup V_k$ is a neighbourhood of a_k, and $V_k \cup V_{k+1}$ is a neighbourhood of b_k. To prove the openness of U it is enough to show that $V_k \subset U$. To this aim we will extend the sequence $\{A_1, \dots, A_m\}$ to a sequence satisfying (11.1)–(11.3) and passing through a given point $y \in V_k$. For an index j such that $0 \leqslant j < m - k$ let $A_{m+j+1} = \widetilde{A}_{m-j}$. Further, let A_{2m-k+1} be the arc contained in A_k with $a_{2m-k+1} = b_k$ and $b_{2m-k+1} = z$. In case when $z = b_k$ we delete the degenerate arc A_{2m-k+1} from our sequence. By Theorem 10 there is an arc A_{2m-k+2} with $a_{2m-k+2} = y$ and $f(b_{2m-k+2}) \in \{0, 1\}$ such that $f|A_{2m-k+2} : A_{2m-k+2} \to [0, 1]$ is an embedding. Therefore the sequence $\{A_1, \dots, A_{2m-k+2}\}$ satisfies conditions (11.1)–(11.3) and passes through y as needed. This finished the proof of the openness of U.

To prove that the set U is closed consider a sequence of points $x_m \in U$ tending to a point $x \in A$. Again by Theorem 10 there is an arc L in A containing the point x and such that $f|L : L \to [0, 1]$ is a homeomorphism. Then there is an index $i \in \mathbb{N}$ and a point $z \in L$ such that $f(z) = f(x_i)$ and $d(z, x_i) < \varepsilon$. Since $x_i \in U$, there is a finite sequence $\{A_1, \dots, A_m\}$ of arcs satisfying conditions (11.1)–(11.3) and passing through x_i. Proceeding as in the proof of the inclusion $V_k \subset U$ (see the previous part) we conclude that there is an extension of the sequence $\{A_1, \dots, A_m\}$ to a sequence $\{A_1, \dots, A_{m'}\}$ such that $y \in A_{i'}$ for some $i' \in \{m + 1, \dots, m'\}$. Therefore $x \in U$, which finishes the proof of the closedness of U. So we have shown that $U = A$. Consequently, there is a finite sequence $\{A_1, \dots, A_n\}$ of arcs satisfying conditions (11.1)–(11.3) and such that $p \in A_k$ for some $k \in \{1, \dots, n\}$. If $p = a_k$, then restricting A_{k-1} and extending A_k slightly we get a sequence of arcs satisfying (11.1)–(11.4). Similarly, if $p = b_k$, then we extend A_k and restrict A_{k+1} to get the needed new sequence.

Note that if a sequence of arcs $\{A_1, \dots, A_{n'}\}$ satisfies (11.1)–(11.4), then the sequence

$$\{A_1, \dots, A_n\} = \{A_1, \dots, A_{n'}, \widetilde{A}_{n'}, \dots, \widetilde{A}_1\}, \quad \text{where } n = 2n',$$

satisfies conditions (11.1)–(11.5). Dividing each arc A_i into arcs of the diameters less than ε we get a sequence of arcs satisfying all the conditions (11.1)–(11.6). This finishes the proof.

Denote by $\mathbb{Q}$ the Hilbert cube, and by $\pi_1 : \mathbb{Q} \to [0,1]$ the projection of $\mathbb{Q}$ onto its first factor.

LEMMA 12. *For each continuum X and for each mapping $f : X \to [0,1]$ there is an embedding $e : X \to \mathbb{Q}$ of X into $\mathbb{Q}$ such that $f = \pi_1 \circ e$.*

Proof. Let $g : X \to \mathbb{Q}$ be any embedding. Define $e : X \to [0,1]\times\mathbb{Q} \simeq \mathbb{Q}$ by $e(x) = \langle f(x), g(x)\rangle$. Then e satisfies the required conditions.

Given a compact space X with a metric d, we let 2^X to denote the hyperspace of all nonempty closed subsets of X equipped with the Hausdorff metric H_X defined by

$$H_X(A,B) = \max\{\sup\{d(a,B) : a \in A\}, \sup\{d(b,A) : b \in B\}\}$$

(see e.g. [16, (0.1), p. 1]).

THEOREM 13. *If there is a confluent mapping $h : B \to [0,1]$ of a continuum B onto the closed unit interval $[0,1]$, then there exists a compactification $X = B \cup C$ of the ray C with B as the remainder, such that there is an open mapping $f : X \to [0,1]$. Moreover, if h is light, then f is light as well.*

Proof. By Lemma 12 we may assume that $B \subset \mathbb{Q}$ and $h = \pi_1|B$. Let $h_1 : B \to h_1(B) = A$ and $h_2 : A \to [0,1]$ be monotone and light mappings, respectively, such that $h = h_2 \circ h_1$, according to Whyburn's monotone-light factorization theorem (see [17, Theorem (4.1), p. 141]). Since h is confluent, we infer that h_2 is light and confluent (see [3, IV, p. 214 and VII, p. 215]). Because a light mapping defined on a compact space is confluent if and only if it is open, provided that the range is locally connected (see [6, Proposition 1, p. 212]), we conclude that h_2 is light open.

Let $\{d_1, d_2, \ldots\}$ be a dense subset of the continuum B. Identifying B with $B \times \{0\}$ in $\mathbb{Q} \times [0,1]$, we will construct a ray $C \subset \mathbb{Q} \times [0,1]$ approximating B.

For the whole proof choose a point $a \in A$ such that $h_2(a) = 0$. Fix $m \in \mathbb{N}$. Let δ_m be a positive number such that for every $x, y \in A$ the condition $d(x,y) < \delta_m$ implies $|h_2(x) - h_2(y)| < \frac{1}{m}$. By Proposition 11 with h_2 in place of f, the point d_m in place of p and with $\varepsilon = \min\{\delta_m, \frac{1}{m}\}$, there is a finite sequence of arcs $A_1^m, \ldots, A_{n_m}^m$ with $A_i^m = a_i^m b_i^m$ for $i \in \{1, \ldots, n_m\}$ satisfying the following conditions:

(13.1) the restrictions $h_2|A_i^m : A_i^m \to [0,1]$ are embeddings for each index $i \in \{1, \ldots, n_m\}$;

(13.2) $a_1^m = a$ and, for each $i \in \{1, \dots, n_m - 1\}$, we have $d(b_i^m, a_{i+1}^m) < \frac{1}{m}$ and $h_2(b_i^m) = h_2(a_{i+1}^m)$;

(13.3) for each $i \in \{1, \dots, n_m - 1\}$ we have either $h_2(A_i^m) \cap h_2(A_{i+1}^m) = \{h_2(b_i^m)\}$ or $h_2(b_i^m) \in \{0, 1\}$;

(13.4) $h_1(d_m) \in A_{k_m}^m$ for some $k_m \in \{1, \dots, n_m\}$ and, if $h(d_m) \notin \{0, 1\}$, then $h_1(d_m) \in A_{k_m}^m \setminus \{a_{k_m}^m, b_{k_m}^m\}$;

(13.5) $b_{n_m}^m = a$;

(13.6) $\operatorname{diam} A_i^m < \delta_m$ for each $i \in \{1, \dots, n_m\}$.

Choose numbers $t_1^m, \dots, t_{n_m}^m \in [\frac{1}{m+1}, \frac{1}{m}]$ such that

$$\frac{1}{m} > t_1^m > t_2^m > \dots > t_{n_m}^m > \frac{1}{m+1}.$$

Denote by $\pi_{\mathbb{Q}} : \mathbb{Q} \times [0, 1] \to \mathbb{Q}$ the projection. For each $j \in \{1, \dots, n_m\}$ let B_j^m be an arc in $\mathbb{Q} \times \{t_j^m\}$ with end points p_j^m and q_j^m such that

$$H_{\mathbb{Q}}(h_1^{-1}(A_j^m), \pi_{\mathbb{Q}}(B_j^m)) < \frac{1}{m}, \quad h_1(\pi_{\mathbb{Q}}(p_j^m)) = a_j^m \quad \text{and} \quad h_1(\pi_{\mathbb{Q}}(q_j^m)) = b_j^m.$$

For each $j \in \{1, \dots, n_m - 1\}$ let C_j^m be an arc in $\mathbb{Q} \times [t_{j+1}^m, t_j^m]$ with its end points $q_j^m \in B_j^m$ and $p_{j+1}^m \in B_{j+1}^m$ such that $\pi_{\mathbb{Q}}(C_j^m) \subset N_{\mathbb{Q}}(B, \frac{1}{m})$ and $C_j^m \setminus \{q_j^m, p_{j+1}^m\} \subset \mathbb{Q} \times (t_{j+1}^m, t_j^m)$. Thus C_j^m is an arc joining the respective end points of B_j^m and B_{j+1}^m. Additionally let $C_{n_m}^m$ be an arc in $\mathbb{Q} \times [t_1^{m+1}, t_{n_m}^m]$ with end points $q_{n_m}^m$ and p_1^{m+1} such that $\pi_{\mathbb{Q}}(C_{n_m}^m) \subset N_{\mathbb{Q}}(h_1^{-1}(a), \frac{1}{m})$ and $C_{n_m}^m \setminus \{q_{n_m}^m, p_1^{m+1}\} \subset \mathbb{Q} \times (t_1^{m+1}, t_{n_m}^m)$.

Define the ray C by

$$C = B_1^1 \cup C_1^1 \cup B_2^1 \cup C_2^1 \cup \dots \cup B_{n_1}^1 \cup C_{n_1}^1 \cup \dots \cup B_1^m \cup C_1^m \cup B_2^m \cup C_2^m \cup \dots \cup B_{n_m}^m \cup C_{n_m}^m \cup \dots .$$

Because of (13.4) we have $d_m \in N_{\mathbb{Q}}(\pi_{\mathbb{Q}}(B_{k_m}^m), \frac{1}{m})$, and by density of $\{d_1, d_2, \dots\}$ we see that $B = B \times \{0\} \subset \operatorname{cl} C$. Therefore $X = B \cup C$ is the needed compactification.

Now we will extend the confluent mapping $h : B \to [0, 1]$ to an open mapping $f : X \to [0, 1]$. For each $m \in \mathbb{N}$ and $j \in \{1, \dots, n_m\}$ put $f(p_j^m) = h(\pi_{\mathbb{Q}}(p_j^m)) = h_2(a_j^m)$ and let $f(q_j^m) \in (h_2(a_j^m), h_2(b_j^m))$ be a point satisfying

$$|f(p_j^m) - f(q_j^m)| = \frac{m}{m+1} \cdot |h_2(a_j^m) - h_2(b_j^m)|.$$

Thus $f : X \to [0, 1]$ is the needed extension of h. The continuity of $f|C$ can be verified by checking the values of f at the end points of B_j^m and C_j^m, of C_j^m and B_{j+1}^m, and of $C_{n_m}^m$ and B_1^{m+1}, respectively. The continuity of f at the points of B is a consequence of the definition of $f|B_j^m$ and the fact that $\operatorname{diam} f(B_j^m) < \operatorname{diam} h_2(A_j^m) < \frac{1}{m}$.

The openness of f is a consequence of the density of the set $\{d_1, d_2, \ldots\}$ in B and the fact that $h(d_m) \in \operatorname{int} f(B^m_{k_m})$ by (13.4).

Observe that $f|C$ is light, whence f is light if h was. The proof is finished.

COROLLARY 14. *The following conditions are equivalent for a continuum B :*

(14.1) *there exists a confluent mapping of B onto* $[0,1]$;

(14.2) *there exists an open mapping of a compactification $X = B \cup C$ of the ray C with B as the remainder onto* $[0,1]$.

In other words, $B \in$ **NC** *if and only if for each compactification $X = B \cup C$ of the ray C with B as the remainder we have $X \in$* **NO**.

Proof. The implication from (14.1) to (14.2) is Theorem 13. The opposite one is Theorem 6.

COROLLARY 15. *The following conditions are equivalent for a continuum B :*

(15.1) *there exists a light open mapping of B onto* $[0,1]$;

(15.2) *there exists a light confluent mapping of B onto* $[0,1]$;

(15.3) *there exists a light open mapping of a compactification $X = B \cup C$ of the ray C with B as the remainder onto* $[0,1]$.

Proof. Conditions (15.1) and (15.2) are equivalent because a light mapping defined on a compact space is confluent if and only if it is open, provided that the range is locally connected (see [6, Proposition 1, p. 212]). The implication from (15.2) to (15.3) is Theorem 13, and the opposite one follows from Proposition 4 because B is a terminal subcontinuum of X and $f(B) = [0,1]$.

MATHEMATICAL INSTITUTE, UNIVERSITY OF WROCŁAW, PL. GRUNWALDZKI 2/4, 50–384 WROCŁAW, POLAND (JJC)

(INSTYTUT MATEMATYCZNY, UNIWERSYTET WROCŁAWSKI)

INSTITUTO DE MATEMÁTICAS, UNAM, CIRCUITO EXTERIOR, CIUDAD UNIVERSITARIA, 04510 MÉXICO, D. F., MÉXICO (JJC)

e-mail: jjc@gauss.matem.unam.mx

DEPARTMENT OF MATHEMATICS AND STATISTICS, UNIVERSITY OF MISSOURI-ROLLA, ROLLA, MO 65409–0020, U. S. A. (WJC)

e-mail wjcharat@umr.edu

DEPARTMENT OF MATHEMATICAL ANALYSIS, CHARLES UNIVERSITY, SOKOLOVSKÁ 83, CZ-186 75 PRAGUE 8, CZECH REPUBLIC (PP)

e-mail pyrih@karlin.mff.cuni.cz

REFERENCES

[1] J. M. Aarts, P. van Emde Boas, *Continua as remainders in compact extensions*, Nieuw Arch. Wisk., **15** (1967) 34–37.

[2] D. E. Bennett, J. B. Fugate, *Continua and their non-separating subcontinua*, Dissertationes Math. (Rozprawy Mat.), **149** (1977) 1–46.

[3] J. J. Charatonik, *Confluent mappings and unicoherence of continua*, Fund. Math., **56** (1964) 213–220.

[4] J. J. Charatonik, W. J. Charatonik, P. Krupski, *Dendrites and light open mappings*, Proc. Amer. Math. Soc., **128** (2000) 1839–1843.

[5] J. J. Charatonik, W. J. Charatonik, S. Miklos, P. Spyrou, *On open mappings of locally connected continua onto arcs*, Houston J. Math., **24** (1998) 21–43.

[6] J. J. Charatonik, K. Omiljanowski, *On light open mappings*, Baku International Topological Conference Proceedings, ELM, Baku 1989, 211–219.

[7] J. J. Charatonik, P. Pyrih, *Continua with light open mappings*, Questions Answers Gen. Topology, **17** (1999) 237–256.

[8] J. J. Charatonik, P. Pyrih, *Atomic mappings can spoil lightness of open mappings*, Tsukuba J. Math., **24** (2000) 157–169.

[9] W. J. Charatonik, *The Lelek fan is unique*, Houston J. Math., **15** (1989) 27–34.

[10] J. B. Fugate, *Decomposable chainable continua*, Trans. Amer. Math. Soc., **123** (1966) 460–468.

[11] G. R. Gordh, Jr., *Terminal subcontinua in hereditarily unicoherent continua*, Pacific J. Math., **47** (1973) 457–464.

[12] K. Kuratowski, *Topology*, vol. 2, Academic Press and PWN, New York, London and Warszawa 1968.

[13] T. Maćkowiak, E. D. Tymchatyn, *Continuous mappings on continua II*, Dissertationes Math. (Rozprawy Mat.), **225** (1984) 1–57.

[14] W. Makuchowski, *On open light mappings*, Comment. Math. Univ. Carolin., **35** (1994) 779–788.

[15] S. B. Nadler, Jr., *Confluent images of the sinusoidal curve*, Houston J. Math., **3** (1977) 515–519.

[16] S. B. Nadler, Jr., *Hyperspaces of sets*, M. Dekker, (1978).

[17] G. T. Whyburn, *Analytic topology*, Amer. Math. Soc. Colloq. Publ., **28**, Providence 1942, reprinted with corrections 1971.