Abstract—This paper proposes a bandwidth-efficient frequency-domain equalization (FDE) for single carrier (SC) underwater acoustic (UWA) communications with multiple transducers and hydrophones. The proposed algorithm implements an overlapped-window FDE by partitioning a large block into small subblocks. A decision-directed channel estimation scheme is also proposed to track the channel variation by the detect symbols. The proposed algorithm is tested by undersea data collected during the Rescheduled Acoustic Communications Experiment (RACE) in March 2008. The experimental results demonstrate the proposed receive estimation scheme is also proposed to track the channel variation by the detect symbols. The proposed algorithm is tested by undersea data collected during the Rescheduled Acoustic Communications Experiment (RACE) in March 2008. The experimental results demonstrate the proposed receive estimation scheme greatly improves the performance in real-world undersea experiments.

I. INTRODUCTION

High data-rate shallow underwater acoustic (UWA) communications have always been challenging due to the adverse effects imposed by hostile underwater propagation environment [1]-[3]. The excessively long multi-path delay spread and frequency-dependent propagation attenuation lead to severe inter-symbol interference (ISI). Besides, relative motion between transceivers and dynamic motion of water surface result in not only time-varying Doppler spread but also fast time-varying channels.

In order to mitigate ISI and Doppler effect, time-domain decision feedback equalization (TD-DFE) with a phase-locked loop (PLL) has been successfully applied in single-input single-output (SISO) [4] and multiple-input multiple output (MIMO) [5] UWA communications. However, due to the fast time-varying fading channels with long channel length, the TD-DFE with PLL is often unstable and less robust to channel variation. And the computational complexity of time-domain equalizers is prohibitive for long delay spread. Therefore, frequency-domain equalization (FDE) which provides lower complexity and better robustness has recently been applied to UWA communications with both multicarrier [6], [7] and single carrier transmission [8], [9], which all exhibit excellent performance in real-world undersea experiments.

However, the challenge of the current frequency-domain (FD) methods (OFDM and SC-FDE) is the conflicting goal of improving bandwidth efficiency and tracking channel variation. Generally, the FD methods require block transmission and zero padding (ZP) or cyclic prefix (CP) [10], which is part of the overhead, has to be inserted between blocks to avoid interblock interference (IBI). For high date-rate UWA communications, the channel memory length is often on the order of a hundred taps, while the channel coherent time only spans about a couple of hundred symbols. The data block length has to be smaller than the coherent time to effectively tracking the channel variation, while the length of the overhead needs to be larger than the channel memory length. Therefore, small block length can track channel variation and ensure good BER performance but suffering from low data efficiency. Increasing data block length may improve bandwidth efficiency but suffering from reduced performance due to poor channel estimation and tracking.

In order to solve the dilemma between the data efficiency and channel tracking, we propose a bandwidth-efficient SC-FDE with a decision-directed channel estimation scheme in this paper. The proposed method employs an overlapped-window for FDE [11], [12] with large block length to simultaneously maintain high bandwidth efficiency and improve capability of channel tracking. In the proposed scheme, the data block with a large length is divided into small subblocks, and a data window consisting of the current subblock and parts of previous and subsequent subblocks is employed to save the input data for the FDE. The symbols of previous and subsequent subblocks are used for precursor and postcursor interference cancellation. The desired equalized subblock data is obtained by discarding the precursor and postcursor parts of the equalized data window. The phase rotation of equalized symbols is corrected by a group-wise phase correction algorithm. The UWA MIMO channels are initially estimated by pilot symbols and re-estimated by the detected subblock symbols in a decision-directed method. The novelty of the proposed FDE scheme lies in that transmission with large block length is allowed in the UWA FDE systems, and the channel impulse responses are estimated and tracked adaptively by the detected symbols. Hence, the proposed scheme greatly improves the bandwidth efficiency and channel tracking capacity at the cost of slightly increased complexity.

The performance of the proposed algorithm has been tested...
by the undersea data collected in the Reschedule Acoustic Communication Experiment (RACE) in Narragansett Bay, Rhode Island from March 1st to March 17th, 2008. This experiment was designed for 400 m and 1000 m ranges, with 2 transducers and 12 hydrophones. The QPSK-modulated signals with a bandwidth of 3.90625 kHz were transmitted over the UWA channels at the carrier frequency 11.5 kHz. Experimental results show that the proposed scheme effectively tracks the time-varying UWA channels, and the average uncoded BER achieves more than 70% reduction of bit error rates over the traditional SC-FDE at the same data efficiency. The transmission overhead of the proposed FDE scheme is only 8.4%, which is significantly reduced compared with more than 20% overhead of other UWA OFDM [6] and SC-FDE systems [8], [14] at the same level of BERs.

Throughout the paper, we use boldface letters to denote vectors and matrices, and the superscripts $^T$, $^H$, $^{-1}$, and $^\dagger$ to denote the matrix transpose, Hermitian transpose, inverse, and pseudo-inverse, respectively.

II. SYSTEM MODEL AND PRELIMINARIES

A MIMO UWA communication system with $N_t$ transducers and $N_r$ hydrophones is considered here. At the transmit end, $N_t$ uncoded bit streams are independently mapped to phase shift keying (PSK) modulated data symbols grouped into data blocks. For the sake of saving transmit power, a gap with zeros, rather than CP, is appended to each block. The zero-padded $N_t$ data streams are transmitted simultaneously and independently over the UWA channels at the same carrier frequency. The receiver structure with $N_r$ hydrophones is shown in Fig. 1, where the received signals are first preprocessed by a front-end component to remove out-of-band noise, synchronize, compensate average Doppler shift, and covert the passband signals to baseband. Next the overlapped-window MIMO FDE is performed to mitigate the ISI and cochannel interference (CCI). The phase rotation caused by Doppler spread is compensated by a group-wise phase rotation algorithm. The UWA channels are initially estimated by known pilot symbols and then tracked by the detected symbols in a decision-directed way.

The equivalent baseband received signals are described in discrete time domain as

$$y_m(k) = \sum_{n=1}^{N_t} \sum_{l=1}^{L} h_{m,n}(l,k)x_n(k-l+1)e^{j(2\pi f_{m,n,k}T_s + \theta_{m,n})} + v_m(k),$$

where $T_s$ is the symbol period, $y_m(k)$ is the received symbol at the $m$-th hydrophone, $v_m(k)$ is the Gaussian noise with an average power of $\sigma^2$, $x_n(k)$ is the transmitted symbol from the $n$-th transducer, $h_{m,n}(l,k)$ is the composite impulse response of the $(m, n)$-th subchannel, $L$ is the channel length, $f_{m,n,k}$ is the time-varying instantaneous Doppler drift, and $\theta_{m,n}$ is the phase error after symbol synchronization.

At the receiver, overlap-adding is performed on each data block before applying FD methods. Then the system model is represented in a matrix format as

$$y_m = \sum_{n=1}^{N_t} D_{m,n} T_{m,n} x_n + v_m$$

where $y_m$, $v_m$ and $x_n$ are the received signal vector, noise vector and transmitted signal vector, respectively, with length $N$. The diagonal matrix $D_{m,n}$ contains phase rotations on its diagonal entries as

$$D_{m,n} = \text{diag}\{e^{j(2\pi f_{m,n,1}T_s + \theta_{m,n})}, \ldots, e^{j(2\pi f_{m,n,N}T_s + \theta_{m,n})}\}$$

and $T_{m,n}$ is the impulse response matrix of size $N \times N$ corresponding to the $(m, n)$-th subchannel. If the block duration $T_b = (N + N_{xp})T_s$ is less than the channel coherence time $\tau_c$, then the time variation of channel impulse response (CIR) is negligible, and $T_{m,n}$ approximates a circulant matrix.

Multiplying normalized DFT matrix $F_N$ on both sides of (2), we obtain the FD representation as

$$Y_m = \sum_{n=1}^{N_t} \Phi_{m,n} H_{m,n} X_n + V_m$$

where $Y_m = F_N y_m$, $X_n = F_N x_n$, $V_m = F_N v_m$, $H_{m,n} = F_N T_{m,n} F_N^H$, and $\Phi_{m,n} = F_N D_{m,n} F_N^H$. The frequency-domain channel response matrix $H_{m,n}$ is diagonal due to the circulant property of $T_{m,n}$, and the $i$-th diagonal component is the frequency response at the $i$-th frequency tone for the $(m, n)$-th channel. Although $\Phi_{m,n}$ is generally a non-diagonal matrix, the diagonal elements of $\Phi_{m,n}$ are significant comparing to the non-diagonal elements if the block duration $T_b$ is less than one third of the quantity $1/\max(f_{m,n,k})$, which is always satisfied because $T_b < \tau_c$ and $\tau_c \ll 1/\max(f_{m,n,k})$ for fixed-to-fixed UWA channels. Hence $\Phi_{m,n}$ is a diagonal-dominant matrix which can be approximated as a diagonal matrix with identical diagonal elements being

$$\Phi_{m,n}(i, i) = \frac{1}{N} \sum_{k=1}^{N} e^{j(2\pi f_{m,n,k}T_s + \theta_{m,n})}, \quad i = 1, 2, \ldots, N.$$
A. Overlapped-Window FDE with Phase Correction

The overlapped FDE is implemented based on the overlap-save method, which is shown in Fig. 2. The data block with a large block size $N$ is divided into small subblocks, and each subblock has a length $N_s$. Thus, there are totally $M = [N/N_s]$ subblocks for one data block. If $N/N_s$ is not an integer, then zeros are padded to the last subblock to make its size to be $N_s$. A overlapped data window is used to form the input data to the equalization. It contains the last $K_1$ points of the previous subblock, the $N_s$ points of the current subblock, and the first $K_2$ points of the subsequent subblock. The $K_1$ points of the previous subblock and the $K_2$ points of the subsequent subblock are saved in the overlapped window to mitigate the precursor and postcursor interference. The size of FFT is $N_f = N_s + K_1 + K_2$. The desired equalized subblock data $x_s^n, n = 1, \ldots, N_t, s = 1, \ldots, M,$ are obtained by discarding the first $K_1$ and the last $K_2$ symbols of the overlapped window.

![Diagram](image)

**Fig. 2.** Data structure for the overlapped-window FDE

We define the estimated channel matrix of the $s$-th subblock in the frequency domain as

$$ \mathcal{H}_s = \begin{bmatrix} \lambda_{1,s}^s \mathbf{H}_{1,1}^s \cdots \lambda_{N_t,s}^s \mathbf{H}_{1,N_t}^s \\ \vdots \\ \lambda_{N_s,s}^s \mathbf{H}_{N_s,1}^s \cdots \lambda_{N_s,N_t,s}^s \mathbf{H}_{N_s,N_t}^s \end{bmatrix}, $$

where $\lambda_{m,n}^s = \frac{1}{N_f} \sum_{k=1}^{N_f} e^{j(2\pi f_{m,n,k} T_t + \theta_{m,n})},$ which is the effect of Doppler and phase errors on the $s$-th subblock of the $(m,n)$-th subchannel, and $\mathbf{H}_{m,n}^s$ is the diagonal frequency response matrix of the $(m,n)$-th subchannel for the $s$-th subblock.

Based on the minimum mean square error (MMSE) criterion, the frequency-domain equalized subblock data is obtained as

$$ \begin{bmatrix} \hat{x}_s^s \\ \vdots \\ \hat{x}_{N_t,n}^s \end{bmatrix} = \mathcal{H}_s^H (\mathcal{H}_s^s \mathcal{H}_s^s + \sigma^2 I_{N_t,N_t})^{-1} \begin{bmatrix} y_1^s \\ \vdots \\ y_{N_t,n}^s \end{bmatrix}, $$

where $y_{m,n}^s$ is the frequency-domain representation of the $s$-th subblock received at the $m$-th hydrophone, and $\hat{x}_s^s$ is the estimate of the frequency-domain representation of the corresponding subblock at the $n$-th transducer. It has been demonstrated [13] that the equalized data $\hat{x}_s^s$ can be represented in frequency domain as

$$ \hat{x}_s^s \approx \sum_{m=1}^{N_r} \Delta_{m,n}^s \Phi_{m,n}^s x_s^n + \hat{v}_s^n, $$

where $\Delta_{m,n}^s$ is approximately a diagonal matrix related to the channel response and equalizer coefficients, and $\Phi_{m,n}^s$ is a diagonal-dominant matrix. Then applying the $N_f$-point IFFT to $\hat{x}_s^n$ yields the time-domain data vector $\hat{x}_n^s$ as

$$ \hat{x}_n^s = \sum_{m=1}^{N_r} \mathbf{F}_{N_f}^H \Delta_{m,n}^s \Phi_{m,n}^s \mathbf{F}_{N_f} x_s^n + \hat{v}_n^s $$

where $x_s^n$ and $\hat{v}_n^s$ are the $T_s$-spaced transmitted signal vector and error vector, respectively. By discarding the first $K_1$ symbols and the last $K_2$ symbols of the equalized overlapped window, the desired equalized symbols of the $s$-th subblock for the $n$-th transducer can be obtained by

$$ \hat{x}_n^s = \hat{x}_n^s(K_1 + 1 : N_f - K_2). $$

Since $\Delta_{m,n}^s \Phi_{m,n}^s$ is a diagonal-dominant matrix and the subblock length is much smaller than the channel coherence time, all the non-diagonal elements of $\mathbf{F}_{N_f}^H \Delta_{m,n}^s \Phi_{m,n}^s \mathbf{F}_{N_f}$ are insignificant comparing to its diagonal elements. Therefore, the $k$-th equalized data symbol in the $s$-th subblock of the $n$-th transducer is expressed by

$$ \hat{x}_n^s(k) = \sum_{m=1}^{N_r} \beta_{m,n}^s|k| e^{j\theta_{m,n}^s|k|} x_s^n(k) + \hat{v}_n^s(k), $$

where $\alpha_{m,n}^s(k) = \sum_{m=1}^{N_r} \beta_{m,n}^s|k|$, $\beta_{m,n}^s$ being the $k$-th diagonal element of the matrix $(\mathbf{F}_{N_f}^H \Delta_{m,n}^s \Phi_{m,n}^s \mathbf{F}_{N_f})$.

From (11), we conclude that the equalized data symbol $\hat{x}_n^s(k)$ is approximately an amplitude-scaled and phase-rotated version of the transmitted data symbol $x_s^n(k)$. When $x_s^n(k)$ is a PSK-modulated symbol, the time-varying rotating phase $\Delta \alpha_{m}^s(k)$ must be compensated before detection.

Since the instantaneous Doppler shift $f_{m,n,k}$ varies gradually over a short period of time, the rotating phase $\Delta \alpha_{m}^s(k)$ also changes slowly and smoothly over time. Therefore, we used an effective and robust group-wise phase correction algorithm [8], [13] to compensate the phase rotations $\Delta \alpha_{m}^s(k)$. In the phase correction algorithm, each equalized subblock is partitioned into $N_g$ groups, and each group has $N_b = N_s/N_g$ symbols.
The initial phase rotation for the subblock is same as the phase rotation of the last group of the previous subblock. The phase correction in each group is conducted in a decision-directed way. The detailed algorithm is omitted here for brevity.

B. Channel Estimation and Tracking

A decision-directed time-domain least square (TD-LS) method is proposed in this paper to estimate and track the channel impulse response (CIRs) of fast-varying UWA channels. The initial channel estimation is obtained by the fixed-length pilot symbols, and the detected subblock symbols are used to re-estimate the channels which are employed to equalize the next subblock. In this method, for the large data block, channel variations are probed by the detected symbols subblock by subblock, rather than inserting more known pilot symbols. Therefore, the overhead of data transmission for channel estimation is greatly reduced, and the variation of channels is effectively tracked in this channel estimation scheme. In our undersea experiment, the data block contains $N = 2048$ symbols, and $N_{zp} = 40$ zeros are padded to each block. The length of pilot symbols $N_p = 150$. Hence, the transmission overhead is $8.4\%$ which is much smaller than many conventional FDE algorithms with $20\%$ pilot overhead [8], [14] and the OFDM transmission with $25\%$ pilot overhead [6], [7] under similar performance.

Let $\hat{x}_n^s$ denote the detected $s$-th subblock of the $n$-th transducer. Then the estimated time-domain CIRs for the $m$-th hydrophone $\hat{h}_m^n = [(\hat{h}_{m,1}^s)^T, \ldots, (\hat{h}_{m,N_p}^s)^T]^T$ can be represented as

$$\hat{h}_m^n = (x^s)^\dagger \cdot y_m^n,$$  \hspace{1cm} (12)

where $x^s = [P_1^s] \cdots [P_{N_p}^s]$ with

$$P_n^s = \begin{bmatrix}
\hat{x}_n^s(L) & \hat{x}_n^s(L-1) & \ldots & \hat{x}_n^s(1) \\
\hat{x}_n^s(L+1) & \hat{x}_n^s(L) & \ldots & \hat{x}_n^s(2) \\
\vdots & \ddots & \ddots & \vdots \\
\hat{x}_n^s(N_p) & \hat{x}_n^s(N_p-1) & \ldots & \hat{x}_n^s(N_p-L+1)
\end{bmatrix}$$  \hspace{1cm} (13)

where $n = 1, \ldots, N_t$, $\hat{h}_{m,n}^s = [\hat{h}_{m,n}^s(1), \ldots, \hat{h}_{m,n}^s(L)]^T$, and $y_m^n = [y_m^s(L), \ldots, y_m^s(N_p)]^T$ which is the corresponding MIMO channel output vector. The initial channel estimation is also obtained by (12) except that known pilot symbols are adopted in (13).

IV. Field Test Results from the RACE08 Experiment

In this section, the proposed bandwidth-efficient receive algorithm is tested by the real-word experimental data. The experimental data was collected during the Rescheduled Acoustic Communications Experiment (RACE) in Narragansett Bay, Rhode Island, conducted by Woods Hole Oceanographic Institution (WHOI), in March 2008. Two receivers were located at 400 meters and 1000 meters away from the transmitter. The receivers were equipped with 12 hydrophones, located 2 meters above the bottom of sea, and the transmitter was mounted with 2 transducers, located 4 meters above the bottom of sea. The water depth varied between 9 to 14 meters. The carrier frequency $f_c = 11.5$ kHz, the sampling rate $f_s = 39.0625$ kHz, and the bandwidth $B = f_s/10 = 3.90625$ kHz. Binary information bits without error correction coding were mapped into QPSK symbols which were grouped into blocks with size of $N = 2048$. A m-sequence with length of 511 was adopted at the beginning of each frame to synchronize the data frames. The received data were downsampled at 2 samples/symbol, thus the $2N_f$-point FFT was employed in the equalizer to improve the BER performance [14].

The length of subblock was set to $N_s = 200$, thus one data block was separated into 11 subblocks. The channels span 25 symbol periods, and we chose $K_1 = K_2 = 40$. The typical time-varying channel impulse responses (CIRs) of one representative data block are shown in Fig. 3 and Fig. 4 for the 400 m and 1000 m system, respectively. These two figures depict the channels estimated by the pilot symbols, the 5-th subblock and the 10-th subblock, respectively. Due to less attenuation caused by the shorter range communications, the amplitudes of CIRs of the 400 m system are larger than that of the 1000 m system. It is seen clearly that even in one data block duration, the amplitude of CIRs are still varying fast. The fast variation of the channels will cause the degradation of performance of the equalizer. Our decision-directed channel estimation method effectively tracks the variations of the channels for both systems and provides more accurate channel estimation for the equalization.

![Fig. 3. The CIRs in one block duration for the 400 m system](image-url)
FDE method with more than 70% reduction of average bit errors for these two range systems.

**TABLE I**

<table>
<thead>
<tr>
<th>Identifier of packet</th>
<th>Traditional FDE method</th>
<th>Proposed FDE method</th>
<th>Error Reduction</th>
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<tr>
<td>0791754F06_C0_S4</td>
<td>0.0856</td>
<td>0.0189</td>
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<td>Avg.</td>
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**Table II**

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V. CONCLUSION

In this paper, a bandwidth efficient FDE with the decision-directed channel estimation was proposed for SC MIMO UWA communications. The proposed receiving algorithm greatly increased the data efficiency of transmissions and significantly improved the system performance. The algorithm has been applied to process undersea data collected during the RACE08 ocean experiments. The time-varying multipath underwater acoustic MIMO channels were tracked well by the decision-directed channel estimation method, and the average uncoded BER of QPSK modulation achieved 1.4% for the 400 m range system and 0.6% for the 1000 m range system with only approximate one third of transmission overhead of traditional FDEs. Compared with traditional FDEs at the same bandwidth efficiency, the proposed algorithm has error reduction rate varying from 63% to 93% at the subblock size $N_s = 200$.

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