ENGINEERING ECONOMY

REVIEW FOR FUNDAMENTALS OF ENGINEERING EXAMINATION

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A. Introduction and Definitions

1. Engineering Economy Study: An evaluation and comparison of alternatives in which differences among those alternatives are expressed as monetary values. Usually involves a decision-making process with these steps:
   a. Recognition and formulation of the problem
   b. Search for feasible alternatives
   c. Quantification and analysis of alternatives
   d. Selection of the best alternative (least cost)

2. Time Value of Money: When making comparisons among alternatives, the cash flows (transactions) are important in terms of their amounts and location in time. Cash flows that occur closer to the present are generally worth more to you than similar dollar cash flows occurring many years in the future.

3. Equivalence: Cash flow amounts that differ in terms of magnitude may have the same value to you depending on their timing and the interest rate involved.

4. Interest: A return, or gain, that may be expected on funds that are productively invested; or cost to borrow money from another source for your own use.

5. Compound Interest: The interest calculation procedure which requires interest to be earned (charged) against the interest already earned as well as the principal. (As opposed to Simple Interest, which does not compute interest on interest).

B. Basic Financial Mathematics

1. Definition of Symbols (P, F, A, G, C, i, and n):
   P = a single “Present” amount occurring at the start of an interest period
   F = a single “Future” amount occurring at the end of an interest period
   A = a series of equal (uniform) amounts occurring at the end of consecutive interest periods
   G = a uniformly increasing (decreasing) series of end-of-period amounts, where successive amounts change by a constant increment from one interest period to the next.
   C = a geometrically increasing (decreasing) series of end-of-period amounts, where successive amounts change by constant percentage multiplier from one interest period to the next.
   i = compound interest rate, per interest period
   n = number of interest periods

2. Cash Flow Diagram: A “time line” sketch showing when cash flows occur, their amount, and whether or not the cash is flowing in or out of a project, investment, or alternative.
3. Standard formulas: Relationships among the previously defined cash flow symbols have been derived for standard configurations, so equations and interest tables are readily available. When using the equations and tables, the cash flows must conform to the standard pattern of cash flows for which the equations were developed. If a situation deviates from a standard pattern, then extra steps are usually necessary in order to solve a problem. Several standard configurations of cash flows for P, F, A, G, and C follow:

\[
P = F(F/F, \ i, \ n)\\
F = P(F/P, \ i, \ n)
\]

\[
F = A(F/A, \ i, \ n)\\
A = F(A/F, \ i, \ n)
\]

\[
P = A(P/A, \ i, \ n)\\
A = P(A/P, \ i, \ n)
\]

\[
P = G(P/G, \ i, \ n)\\
G = P(G/P, \ i, \ n)
\]

When \( r = i \)

\[
P = \frac{C \cdot n}{(1+r)^n-1}
\]
4. Examples of Standard Form Applications:

a. What amount must be invested today in order to have $40,000 available 10 years from now, if interest is 10 percent?

\[ P = F \left( \frac{P}{F}, 10\%, 10 \right) \]
\[ = 40K \times 0.3855 \]
\[ = 15,420 \]

b. If $5,000 is placed in an account that earns 6.75% interest, how much will that account be worth in 28 years?

\[ F = P \left( 1 + i \right)^n \]
\[ = 5K \times (1.0675)^{28} \]
\[ = 31,136.57 \]

c. How much will be accumulated in a fund earning 8 percent interest if $1,000 is deposited in that fund at the end of the year for the next 12 years?

\[ F = A \left( \frac{F}{A}, 8\%, 12 \right) \]
\[ = 1000 \times 18.9971 \]
\[ = 18,997.10 \]

d. An individual needs to have $5,000 for a down payment to be placed on a tract of land 15 months from now. If she has access to a fund that earns 1.00 percent interest per month, what uniform amount must be placed in that fund starting one month from now?

\[ A = F \left( \frac{A}{F}, 1\%, 15 \right) \]
\[ = 5K \times 0.06212 \]
\[ = 310.60 \]
e. An account which earns 8 percent interest currently has a balance of $10,000. How much can be withdrawn from this account at the end of each year for the next 7 years and have the account depleted at the time of the last withdrawal?

\[
P = \frac{A}{P(8\%, 7)}
\]

\[
A = 10,000 \times (0.1921) = 1921.20
\]

f. What amount would you need to deposit at 8 percent interest on January 1, 2003, in order to be able to draw out $2,000 at the end of each year for the next 8 years, leaving the fund depleted at the end of that time?

\[
P = A \times (P/A, 8\%, 8)
\]

\[
2000 = 11,493.20
\]

g. Repair costs on a new pump are estimated to be $0 at the end of the first year; $25 at the end of the second year; $50 at the end of the third year, $75 at the end of the fourth year and continuing in this manner until the end of the tenth year. If interest is 8 percent, find the present worth equivalent of these maintenance costs.

\[
P = G \times (P/G, 8\%, 10)
\]

\[
= 25 \times 25.9768 = 649.42
\]

h. A geometric gradient with \( r = 10\% \) per year has an E0Y1 value equal to 100. What is the present worth equivalent of this gradient if interest is 10 percent and \( n = 5 \) years?

\[
P = Cn / (1+r)
\]

\[
= 100 \times (5) / (1.10) = 454.55
\]
C. Problems having the number of compounding periods "n" as the unknown.

1. Direct Solution with exponents or logarithms.

\[ P \ (1 + i)^n = F \quad \Rightarrow \quad n = \log \frac{F}{P} / \log (1 + i) \]

**EXAMPLE:** How many years must $2,000 be invested at 8 percent in order to reach a value of $4,000?

\[ n = \log \frac{4,000}{2,000} / \log (1 + .08) = 0.30103 / 0.033424 = 9.006 \approx 9 \text{ years} \]

2. Search (interpolate) for tabulated interest factor. When using the more complex interest factors, the best method is to calculate the value of the interest factor and attempt to locate it in the appropriate interest table, thus determining the number of interest periods.

**EXAMPLE:** A deposit of $1,000 is now placed in an account earning 8 percent interest. How many successive end-of-year withdrawals equal to $98 each may be transacted before the account is depleted?

Use: 1,000 \((A/P, 8\%, n = ?) = 98\) \((A/P, 8\%, n = ?) = 0.098\)

Look in 8.00% Interest Factor Table under \((A/P)\) factor for 0.098: \(n = 22 \text{ years}\)

D. Problems having the interest rate "i" as the unknown.

1. Direct solution with exponents:

\[ P \ (1 + i)^n = F \quad \Rightarrow \quad i = \left(\frac{F}{P}\right)^{1/n} - 1 \]

**EXAMPLE:** A deposit of $1,000 increases in value to a sum of $2,500 in just 5 years. What compound interest rate was earned?

\[ i = \left[\frac{2,500}{1,000}\right]^{1/5} - 1 = 1.2011 - 1 = 0.2011 = 20.11\% \]

2. Search (interpolate) for tabulated interest factor. May involve a search between interest tables. The interest factor must first be calculated; then attempt to locate it in the tables using the pertinent "n" and correct column.

**EXAMPLE:** A series of 10 annual end-of-year deposits equal to $500 each were placed in an account. The account was worth $7,600 just after the last deposit was made. What interest rate was earned in this situation?

Use: 500 \((F/A, i = ?, 10) = 7,600\) Then find: \((F/A, i = ?, 10) = 15.2\)

<table>
<thead>
<tr>
<th>(i) ((F/A, i, 10))</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.00 % (14,4866)</td>
</tr>
<tr>
<td>? (15.2)</td>
</tr>
<tr>
<td>10.00 % (15,9374)</td>
</tr>
</tbody>
</table>

Linear Interpolation gives: \(i = 8.98\%\)
E. Interest Compounding More Frequent than Annual (Non-Annual Compounding):

If interest is compounded at a consistent interval, shorter than a year, the following terms must be defined:

The compounding interval and the number of intervals in one year, \( m \).

The "Nominal Interest Rate" per year, \( r \) (sometimes shown as \( i_n \)).

The "Effective Interest Rate" per year, \( i_{\text{eff}} \).

\[
i_{\text{eff}} = \left(1 + \frac{(r/m)}{m}\right)^m - 1
\]

The interest rate which applies to the interest interval: \( i = \frac{r}{m} \)

**EXAMPLE**: A deposit of $500 is placed in an account yielding 18% nominal interest compounded monthly. What is this account worth at the end of 6 months; and what was the effective interest rate which was earned?

\[
i = \frac{r}{m} = \frac{0.18}{12} = 0.015
\]

\[
F = 500 \left(1 + 0.015\right)^6 = 500 \left(1.0934\right) = $546.72
\]

\[
i_{\text{eff}} = \left(1 + (0.18/12)\right)^{12} - 1 = \left(1.015\right)^{12} - 1 = 0.1956 = 19.56\%
\]

A special type of very frequent compounding is when the compounding interval becomes so short that there are an infinite number of them within a year. Thus "\( m \)" becomes infinitely large. This is referred to as "Continuous Compounding" and we have a different set of equations using "\( e \)" the base of natural logarithms to be applied. The Continuous Compounding rate is given the symbol "\( r \)."

**EXAMPLE**: Five end-of-year deposits each equal to $800 are placed in an account which yields 10% compounded continuously. What is this account worth at the end of 5 years, immediately after the last deposit?

\[
F = A \left(F/A, \ r\%, \ 5\right) = 800 \left[e^{(r \times n)} - 1\right] / \left[e^r - 1\right]
\]

\[
= 800 \left[e^{0.10 \times 5} - 1\right] / \left[e^{0.10} - 1\right]
\]

\[
= 800 \left[1.64872 - 1\right] / \left[1.10517 - 1\right]
\]

\[= 800 \left(6.16825\right) = $4,934.60\]
F. Present Worth Evaluations and Comparisons

1. To evaluate any configuration of future costs in terms of their present worth equivalent, each component of those costs needs to be "discounted" to the point in time chosen to be the present for the problem.

EXAMPLE: Find the present worth equivalent of a series of $100 payments which starts at the end-of-year 21 and terminates at the end of year 35. The interest rate is 8% compounded annually.

\[ P = 100 \cdot (P/A, 8\%, 15) \cdot (P/F, 8\%, 20) = 800(8.5595)(0.2145) = 1,468.81 \]

2. To compare alternatives, whatever the configuration of costs and/or revenues, there must be consistency in the number of years of service offered by those alternatives. Often it is necessary to assume an identical replacement is available in order to make an alternative having a short service life be compatible with an alternative having a longer service life.

EXAMPLE: Identify the most economical pump to buy of the two candidates listed below. Assume identical replacements will be available in the future if needed. The interest rate is 10 percent.

<table>
<thead>
<tr>
<th>Tiger Pump</th>
<th>Miner Pump</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Cost, $</td>
<td>7,000</td>
</tr>
<tr>
<td>Annual Op. Cost, $</td>
<td>2,000</td>
</tr>
<tr>
<td>Salvage, $</td>
<td>1,500</td>
</tr>
<tr>
<td>Service Life, yrs.</td>
<td>4</td>
</tr>
</tbody>
</table>

\[ PW \text{ (Two Tigers)} = 7000 + (7000 - 1500)(P/F, 10\%, 4) - 1500(P/F, 10\%, 8) + 2000(P/A, 10\%, 8) \]
\[ = 20,727 \]

\[ PW \text{ (One Miner)} = 10000 - 3000(P/F, 10\%, 8) + 1700(P/A, 10\%, 8) \]
\[ = 17,670 \]

Select a Miner Pump since it has the lowest Present Worth of Costs for 8 years!

G. Capitalized Cost

Capitalized Cost evaluation or comparison is a special case of present worth analysis where the service life or project life is assumed to be infinitely long. The time value of money calculations are based on the use of the following:

\[ CC = P = A \cdot (P/A, i, n = \infty) = A \cdot (1 / i) \]

If any configuration of costs is presented, it is necessary to discount all the single value costs, the series costs, and intermittent costs to the present time in order to find the Capitalized Cost. If alternatives are being compared, the assumption of perpetual service assures the same number of years of service will be obtained from the alternatives being considered.
Capitalized Costs:

EXAMPLE: Find the Capitalized Cost for a project having an initial cost of $500,000 and annual maintenance expenses of $10,000. Interest is 8%.

\[ CC = 500,000 + 10,000 / (0.08) = 525,000 \]

EXAMPLE: What is the Capitalized Cost associated with a specific major repair expense that will be $100,000 every 5 years, with the first expense occurring at the end-of-year 5. Interest is 8%.

\[ CC = 100,000 (A/F, 8\%, 5) / (0.08) = 213,125 \]

H. Equivalent Uniform Annual Cost (EUAC) Evaluations and Comparisons

1. To evaluate any configuration of amounts in terms of their uniform annual equivalent, each amount must be converted to an annual end-of-year value, with all of the resulting end-of-year values being equal. This is like finding an "annual average" cost, but it is done taking into account the time value of money. One common situation is where an item is purchased for an initial amount (P value), but is financed with payments (A value) that are uniform and at a regular interval.

EXAMPLE: A vehicle costs $14,000 when new and has a trade-in value of $6,000 at the end of 4 years. Its operating expense is $3,500 per year, with an additional $1,000 for tires, tune up, and battery at the end of the 2nd year. What is the EUAC for owning this vehicle 4 years, assuming interest at 10%.

\[
EUAC = 14,000(A/P, 10\%, 4) - 6000(A/F, 10\%, 4) + 3,500 \\
+ 1,000(P/F, 10\%, 2)(A/P, 10\%, 4) = 6,885
\]

EXAMPLE: A property is purchased for $100,000 now, with this amount being financed for 15 years at 8% interest. What is the annual end-of-year payment?

Annual Payment = 100,000 (A/P, 8\%, 15) = 11,680

2. When comparing alternatives using the EUAC method, for each alternative all of the pertinent costs and revenues must be converted to equal end-of-year values. If the alternatives have different service lives, it is often assumed that identical replacements will be available in the future ... thus the first item in a series of assumed identical replacements is all that need be evaluated.

EXAMPLE: Compare the Tiger Pump and Miner Pump by EUAC

\[
EUAC (Tiger) = 7000(A/P, 10\%, 4) - 1500(A/F, 10\%, 4) + 2000 = 3,885
\]

\[
EUAC (Miner) = 10000(A/P, 10\%, 8) - 3000(A/F, 10\%, 8) + 1700 = 3,312
\]
I. EUAC of Perpetual Service

The Equivalent Uniform Annual Cost of Perpetual Service is a special case of annual cost analysis where the service life or project life is assumed to be infinitely long. To evaluate any configuration of future costs in terms of their annual equivalent for an infinite time, it is often best to discount the set of costs to be a Capitalized Cost, then use the relationship: \( A = CC x (i) \)

EXAMPLE: Find the EUAC of perpetual service for a railroad right of way which has a present value of $1,800,000 if interest is 10 percent compounded annually.

\[
EUAC = 1,800,000 \times (0.10) = 180,000 \text{ per year}
\]

EXAMPLE: Find the EUAC of perpetual service for a series of maintenance expenses which is $100,000 every five years. Interest is 8%.

\[
EUAC = \left[100,000 \times (A/F, 8\%, 5) / (0.08)\right] (0.08) = 17,050
\]

EXAMPLE: Find the EUAC of perpetual service for a set of construction costs which were: EOY 1 = 200,000; EOY 2 = 700,000; EOY 3 = 300,000. i = 8%.

\[
EUAC = [200K(P/F, 8\%, 1)+700K(P/F, 8\%, 2)+300K(P/F, 8\%, 3)] (0.08) = 81,874
\]

J. Future Worth Evaluations and Comparisons

1. To evaluate any configuration of costs in terms of their future worth equivalent, each component of those costs must be "compounded" to the future point in time chosen for the problem.

EXAMPLE: Find the future worth equivalent at the end-of-year 25 of a series of $1,000 deposits made from end-of-year 1 through 15. How much of that future total is interest? The interest rate is 10%.

\[
FW = 1,000(F/A, 10\%, 15)(F/P, 10\%, 10) = 82,408
\]

Interest portion of this total: \( 82,408 - (15)(1,000) = 67,408 \) is interest

2. To compare alternatives by Future Worth evaluation, there must be consistency among the alternatives in the number of years of service offered by each of them.

EXAMPLE: Compare the Tiger Pump and the Miner Pump by FW evaluation.

\[
FW (Two Tigers) = 7,000(F/P, 10\%, 8) + (7,000-1,500)(F/P, 10\%, 4) - 1,500 + 2000(F/A, 10\%, 8) = 44,430
\]

\[
FW (One Miner) = 10,000(F/P, 10\%, 8) + (1,700)(F/A, 10\%, 8) - 3,000 = 37,877
\]

Select a Miner Pump since it has the lowest Future Worth of Costs for 8 years.
K. Benefit – Cost Analysis (Often noted as: Benefit/Cost Ratio)

The Benefit/Cost (B/C) Ratio identifies the benefits (often determined as savings or cost reductions) to be realized per each dollar that is expended to achieve those benefits. Several aspects of setting up the B/C Ratio are:

1. All dollar values used in the B/C calculations must be expressed consistently, such as all annual amounts, or all present worth amounts.

2. Projects, or alternatives, must be tested incrementally, proceeding from the one having the least cost to the one having the greatest cost to the public agency (this assures the denominator will not be a negative value).

3. B/C Ratio less than 1.0 indicates the more costly project is unsatisfactory.
   - B/C Ratio equal 1.0 indicates the more costly project is marginal.
   - B/C Ratio greater than 1.0 indicates the more costly project is acceptable.

EXAMPLE: The Annual Benefits and Annual Costs have already been calculated for 4 alternatives shown below. Determine which is the best alternative using the B/C Ratio procedures.

<table>
<thead>
<tr>
<th>ALT.</th>
<th>ANNUAL BENEFITS</th>
<th>ANNUAL COSTS</th>
<th>WITHIN ALT. B/C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>182,000</td>
<td>91,500</td>
<td>1.99</td>
</tr>
<tr>
<td>A2</td>
<td>167,700</td>
<td>79,500</td>
<td>2.10</td>
</tr>
<tr>
<td>A3</td>
<td>115,000</td>
<td>78,500</td>
<td>1.46</td>
</tr>
<tr>
<td>A4</td>
<td>95,000</td>
<td>50,000</td>
<td>1.90</td>
</tr>
</tbody>
</table>

Although Alternative A2 has the highest within project B/C ratio (2.10), A2 must be also be proven best in an incremental comparison involving all projects, as:

\[
\text{B/C (A3 vs. A4)} = \frac{(115,000 - 95,000)}{(78,500 - 50,000)} = 0.70
\]

A3 is not accepted, next incremental test is A2 vs. A4

\[
\text{B/C (A2 vs. A4)} = \frac{(167,700 - 95,000)}{(79,500 - 50,000)} = 2.46
\]

A2 is accepted, but must defend itself against A1

\[
\text{B/C (A1 vs. A2)} = \frac{(182,000 - 167,700)}{(91,500 - 79,500)} = 1.19
\]

A1 is accepted.

Thus, the Incremental B/C Ratio analysis indicates A1 is the best choice.
L. Valuation and Depreciation

From the perspective of economic analysis, depreciation is an accounting practice which affects the amount of taxes a profit-making organization pays. Depreciation views the cost of an asset as a prepaid expense that is to be charged against profits over a certain number of years. Allowed depreciation procedures are specified by the IRS. The "Straight Line" method could be specified to simplify computations, or the Modified Accelerated Cost Recovery System may be specified since it allows larger depreciation deductions earlier in the life of the asset. The "Book Value" of an asset is its original acquisition cost less its accumulated depreciation costs.
(Notes: The IRS does not allow LAND to be depreciated).

Straight Line Depreciation provides for the depreciation charge to be the same each year for the life of the asset and is based on the equation:

\[ D_j = (C - S) / n \]

for the depreciation charge in year "j"

EXAMPLE: Given \( C = \$18,000 \) \( S = \$3,000 \) \( n = 3 \) years

\[ D_j = (18,000 - 3,000) / 3 = \$5,000 \]

each year

The depreciation charge and book value pattern:

<table>
<thead>
<tr>
<th>YEAR</th>
<th>START OF YEAR BOOK VALUE</th>
<th>DEPRECIATION CHARGE</th>
<th>END OF YEAR BOOK VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18,000</td>
<td>5,000</td>
<td>13,000</td>
</tr>
<tr>
<td>2</td>
<td>13,000</td>
<td>5,000</td>
<td>8,000</td>
</tr>
<tr>
<td>3</td>
<td>8,000</td>
<td>5,000</td>
<td>3,000</td>
</tr>
</tbody>
</table>

Modified ACRS Depreciation is based on a set of IRS tabled percentage factors that are applied to the initial cost of the asset. Salvage value is ignored when computing annual ACRS depreciation charges, however any subsequent gain on a sale of item would be subject to a gains tax. The IRS sets up Recovery Period Classes for assets, and those classes specify the duration of time for allowing depreciation, as well as the specific percentages to apply. For instance, "Special Tools" belong to a 3-Year Recovery Period, however the actual depreciation extends over 4 years, with allowed annual percentages being: 33.3%; 44.5%; 14.8%; and 7.4% respectively.

EXAMPLE: 3 year recovery period class, Initial Cost = \$16,000 \ Salvage = \$3,000

<table>
<thead>
<tr>
<th>YEAR</th>
<th>MACRS %</th>
<th>START OF YEAR BOOK VALUE</th>
<th>DEPRECIATION CHARGE</th>
<th>END OF YEAR BOOK VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>33.3</td>
<td>16,000</td>
<td>5,328</td>
<td>10,672</td>
</tr>
<tr>
<td>2</td>
<td>44.5</td>
<td>10,672</td>
<td>7,120</td>
<td>3,552</td>
</tr>
<tr>
<td>3</td>
<td>14.8</td>
<td>3,552</td>
<td>2,368</td>
<td>1,184</td>
</tr>
<tr>
<td>4</td>
<td>7.4</td>
<td>1,184</td>
<td>1,184</td>
<td>0</td>
</tr>
</tbody>
</table>
M. Break-Even Analysis and Payback Period

A large percent of the data used in an economy study are subject to uncertainty since they are future estimates. A break-even analysis may be performed to specify a range of conditions for which one or the other alternative is preferred. If a break-even point can be determined, it may be possible to predict on which side of the break-even point operations are most likely to occur. A special type of break-even evaluation is the amount of time necessary to recapture the amount of an initial investment. This is referred to as a payback period analysis.

EXAMPLE (Break-Even on hours of operation)

Two electric motors are being compared for possible purchase, with costs:

<table>
<thead>
<tr>
<th></th>
<th>Royal Elec., Co.</th>
<th>Cardinal Power, Inc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Cost</td>
<td>$1,600</td>
<td>$1,250</td>
</tr>
<tr>
<td>Annual Maint. Cost</td>
<td>$ 50</td>
<td>$ 60</td>
</tr>
<tr>
<td>Salvage Value</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Service Life (yrs)</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Efficiency</td>
<td>92 %</td>
<td>85 %</td>
</tr>
</tbody>
</table>

Each motor is rated at 30 horsepower output and electric power costs are estimated at $ 0.05 per kilowatt hour. Interest is 10%. At what number of hours of running time per year would there be no difference between these two electric motors?

ROYAL: Annual equivalent of first cost: 
1600 (A/P, 10%, 10) = 260.32
Annual maintenance: 50.00
Power $/yr: (30 hp/0.92)(0.746kw/hp)($0.05/kwhr)(n hours) = 1.216(n)

CARDINAL: Annual equivalent of first cost: 
1250(A/P, 10%, 10) = 203.38
Annual Maintenance: 60.00
Power $/yr: (30hp/0.85)(0.746kw/hp)($0.05/kwhr)(n hours) = 1.3165(n)

EUAC(Royal) = EUAC(Cardinal): 310.32 + 1.2163(n) = 263.38 + 1.3165(n)

Solve Break-even equation for: n = 468.5 hours of running time

EXAMPLE (Break-Even by Payback Period Analysis)

Which alternative would be preferred according to a Payback Period analysis with i = 0?

Project Sooner: Initial Cost = $600  Annual Profits = $200; 200; 300; 400; 400
Project Later: Initial Cost = $900  Annual Profits = $100; 100; 200; 200; 300; 500

Project Sooner initial cost payback of $600 is achieved in just 3 years.
Project Later initial cost payback of $900 is achieved in 5 years.
N. Bonds

A bond is a financial document (a promise to pay) which has a face value (principal, redemption value, or maturity value) as well as a stated percentage which relates to the amount that is to be paid to the bond buyer at regular intervals (usually 6-month intervals). Specifically, these regular payments are fixed by the stated nominal interest for the bond applied to the face value. The actual market price of a bond at the time it is sold varies with the current market interest rate. Pricing a bond to obtain a desired yield or rate of return involves use of 2 interest rates; namely, the bond interest rate used to determine the amount of the semi-annual payments; and the interest rate related to the overall investment.

EXAMPLE: A $5,000 Municipal Bond bearing 7% interest payable semi-annually matures 10 years hence. What should the present price of this bond be set at in order to offer the buyer an 8% rate of return if purchased now?

Bond interest payments at 6-month intervals: \((0.07/2)(5,000) = 175\)  \(n = 20\)

The rate of return analysis will be based on \((0.08/2) = 0.04\) since it is understood that the 8% rate of return been sought is a nominal interest rate of return.

Bond Price: \(P = 175(P/A, 4\%, 20) + 5000 (P/F, 4\%, 20) = 4,660.20\)

O. Inflation

Inflation causes a "loss in purchasing power" of money over a period of time. There are many indexes published which indicate the extent of the inflation trend in the US. For instance the CPI has a base of 100 for "All Urban Consumers" in 1982 – 84. By 2000 it was 172.2. In engineering economy studies the effects of inflation are often ignored since these studies focus on the differences among alternatives, and it is assumed that inflation would affect all pertinent costs in the same manner ... thus inflation would have no impact on the selection of the best alternative from a set.

When the rate of inflation is considered in an economy study it usually is given the symbol "f", in percent/year. A "real interest rate" is defined as "i" which measures the true growth in purchasing power. A "combined interest rate" is defined as "d" which has the combined effect of real growth and inflation-driven growth. Thus:

\[(1 + d) = (1 + i)(1 + f)\]

and it follows that: \(d = f + i + (i \times f)\)

Most economy studies state cash flows in terms of "current dollars", that is, in dollars in the amount that actually occurred at the time of the transaction. When performing an analysis using "current dollars" the combined interest rate "d" should be used; if the "current dollars" have been adjusted for inflation to become "constant worth dollars", then the real interest rate "i" should be used.

EXAMPLE: $5,000 invested in 1990 increased in value to $9,000 by year 2000.
A. What combined interest rate was earned? \(5000(1+d)^{10} = 9000; \) \(d = 6.05\%\)
B. Suppose that during this same period the inflation rate was \(f = 2.80\%\) per year.
What was the real interest rate for this investment?
\(0.0605 = 0.0280 + i + (i \times 0.0280); \quad i = 3.16\%\) per year
FORMULAS FOR CALCULATING COMPOUND INTEREST FACTORS

**DISCRETE COMPOUNDING with DISCRETE PAYMENTS**

Single Payment
- Compound Amount Factor: \((F/P, i\%, n)\)
- Present Worth Factor: \((P/F, i\%, n)\)

Uniform Series
- Sinking Fund Factor: \((A/F, i\%, n)\)
- Compound Amount Factor: \((F/A, i\%, n)\)
- Capital Recovery Factor: \((A/P, i\%, n)\)
- Present Worth Factor: \((P/A, i\%, n)\)

Arith Gradient
- Future Amount Factor: \((F/G, i\%, n)\)
- Series Conv. Factor: \((A/G, i\%, n)\)
- Present Worth Factor: \((P/G, i\%, n)\)

**CONTINUOUS COMPOUNDING (CC), DISCRETE PAYMENTS**

Single Payment
- CC Future Amount Factor: \((F/P, e^{i\%}, n)\)
- CC Present Worth Factor: \((P/F, e^{i\%}, n)\)

Uniform Series
- CC Sinking Fund Factor: \((A/F, e^{i\%}, n)\)
- CC Future Amount Factor: \((F/A, e^{i\%}, n)\)
- CC Capital Recovery Factor: \((A/P, e^{i\%}, n)\)
- CC Present Worth Factor: \((P/A, e^{i\%}, n)\)

**MODIFIED ACRS FACTORS**

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**Factor Table - i = 8.00%**

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**Factor Table - i = 10.00%**

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