CS 253: Algorithms

Syllabus

Chapter 1

Appendix A
Syllabus

- **Instructor**: Fikret Ercal  -  Office: CS 314  -  Phone: 341-4857
- **E-mail & URL**: ercal@mst.edu  -  http://web.mst.edu/~ercal/index.html
- **Meeting Times**: 1 – 1:50 pm.  M W F
- **Office Hours**: Posted on the class website
  
  **If there is no prior notice and the instructor is late for the class, students are expected to wait ~8 minutes before they leave the classroom.**
- **Grader**: see the CS-253 homepage for the name and the e-mail information
- **Prerequisites**: CS 128 and CS 153
- **Objectives**: 
  - Design algorithms, analyze algorithms for computational efficiency, space, and correctness.
  - Develop strategies for dynamic programming and greedy algorithms, design and study fundamental data structures and algorithms including (but not limited to) heaps, sorting, searching, graph algorithms, hashing, and data compression.
Class Policies

- Class notes (in PPT), syllabus, homework assignments, announcements, and other related materials can be accessed on the class website. Make sure that you regularly check these sites for announcements and course related materials.

- Students are expected to attend all classes unless they have a reasonable excuse for being absent. When in class, you are expected to turn off all pagers, phones, and beepers.

- Academic Alert System (http://academicalert.mst.edu/).

- Projects and homework must be an individual effort unless stated otherwise. Assignments which are unusually similar will receive a zero (0) grade.

- Any student inquiring about academic accommodations because of a disability will be referred to Disability Support Services (http://dss.mst.edu/) so that appropriate and reasonable accommodative services can be determined and recommended.

- No late homework or project will be accepted.
The Role of Algorithms in Computing

- **Algorithm**
  
a sequence of computational steps that transform the *input* (a set of values) into the *output*. 
a tool for solving a well-defined computational problem

**Example:** Sorting Problem

**Input:** A sequence of \( n \) numbers \( a_1, a_2, \ldots, a_n \)

**Output:**

A permutation (reordering) \( a'_1, a'_2, \ldots, a'_n \) of the input sequence such that \( a'_1 \leq a'_2 \leq \cdots \leq a'_n \)

- An algorithm is said to be **correct** if, for every input instance, it halts with the correct output
What kind of Problems are Solved by Algorithms?

- **Human Genome Project**
  3 billion nucleotides. 100K genes are identified using various algorithms including approximate string matching, searching, sorting, alignment, hashing, clustering, etc. etc.

- **Enabling Internet**
  search engines
  network routing algorithms
  cryptography / network security
  …

- **Optimal Use of Resources in Business and Life**
  Examples: optimal placement of oil wells to maximize output, flight scheduling, truck routing, bin packing, ad placement for political campaigns, chip placement for minimal routing, etc. etc.
  *Linear Programming*

- **Developing Strategies to Solve Puzzles, Play Mind Games etc.**
  computer chess, othello, chinese checkers
  *AI Algorithms*
  …
Which one is more important:
advances in hardware speed or improvement in algorithmic complexity?

Computer A: \(10^{11}\) instructions / sec.
Computer B: \(10^8\) instructions / sec.

A is 1000 times faster than B

Insertion Sort: \# of instructions to be executed in order to sort n items = \(2n^2\)

Merge Sort: \# of instructions to be executed in order to sort n items = \(50n \log_{10} n\)

\(n = 1\) million = \(10^6\) items

If A uses Insertion sort and B uses merge sort, which one runs faster?

\(\text{Time}_A = \left[\frac{2\times(10^6)^2}{10^{11}}\right] = 20\) seconds

\(\text{Time}_B = \left[\frac{50\times(10^6) \times \log_{10} (10^6)}{10^8}\right] = 3\) seconds!
**here Appendix A**

**Summations**

\[ \sum_{k=1}^{n} k = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2} \]

\[ \sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6} \]

\[ \sum_{k=1}^{n} k^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4} \]

\[ \sum_{k=0}^{n} x^k = 1 + x + x^2 + \cdots + x^n = \frac{x^{n+1} - 1}{x - 1} \]

If \( |x| < 1 \) \( \Rightarrow \) \( \lim_{n \to \infty} \sum_{k=0}^{n} x^k = \sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \)
Harmonic Series

\[ H_n = \sum_{k=1}^{n} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} = \ln n + O(1) \]

\[ \sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \quad \text{for } |x| < 1 \quad (\text{shown earlier}) \]

Take the derivative of both sides and also multiply with \( x \)

You get the following series summation:

\[ \sum_{k=0}^{\infty} kx^k = \frac{x}{(1-x)^2} \quad \text{for } |x| < 1 \]
Proof by Induction

Claim: $S(n)$ is true for all $n \geq m$

Proof:

- **Basis**: Show formula is true for $n = m$

- **Inductive hypothesis**: Assume formula is true for an arbitrary $n$

- **Step**: Show that formula is then true for $n+1$
Induction Example 1:

Prove that \[ \sum_{k=1}^{n} k = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2} \] for \( n \geq 1 \)

- **Basis:** If \( n = 1 \), then \( 1 =? 1(1+1)/2 \) Yes, TRUE

- **Inductive hypothesis:**
  Assume formula is true for \( n \rightarrow 1 + 2 + 3 + \ldots + n = n(n+1)/2 \)

- **Step (Show that formula is then true for \( n+1 \)):**

  \[
  1 + 2 + 3 + \cdots + n + (n+1) = \frac{(n + 1)((n + 1) + 1)}{2}
  \]

  \[
  \frac{n(n+1)}{2} + (n+1) = \frac{n(n+1) + 2(n+1)}{2} \implies \frac{(n + 1)(n + 2)}{2}
  \]
Induction Example 2:

Prove that \( \sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6} \)

- **Basis:** If \( n = 1 \), then \( 1^2 = 1 \) \( \frac{1(1+1)(2*1+1)}{6} = \frac{3}{6} \) Yes, TRUE

- **Inductive hypothesis:** Assume formula is true for \( n \)

- **Step** (Show that formula is then true for \( n+1 \)):

\[
1^2 + 2^2 + 3^2 + \cdots + n^2 + (n+1)^2 = \frac{(n+1)(n+2)[2*(n+1)+1]}{6}
\]

\[
\frac{n(n+1)(2n+1)}{6} + (n+1)^2 = \frac{n(n+1)(2n+1) + 6(n+1)^2}{6}
\]

\[
\Rightarrow \frac{n(n+1)(2n+1) + 6(n+1)^2}{6} = \frac{(n+1)[n(2n+1) + 6(n+1)]}{6}
\]

\[
\Rightarrow \frac{(n+1)[n(2n+1) + 6(n+1)]}{6} = \frac{(n+1)(2n^2 + 7n + 6)}{6} = \frac{(n+1)(n+2)(2n+3)}{6}
\]