Tensor

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Tensors are geometric objects that describe linear relations between vectors, scalars, and other tensors. Elementary examples of such relations include the dot product, the cross product, and linear maps. Vectors and scalars themselves are also tensors. A tensor can be represented as a multi-dimensional array of numerical values. The order (also degree or rank) of a tensor is the dimensionality of the array needed to represent it, or equivalently, the number of indices needed to label a component of that array. For example, a linear map can be represented by a matrix, a 2-dimensional array, and therefore is a 2nd-order tensor. A vector can be represented as a 1-dimensional array and is a 1st-order tensor. Scalars are single numbers and are thus zeroth-order tensors.

Tensors are used to represent correspondences between sets of geometrical vectors. For example, the stress tensor T takes a direction v as input and produces the stress T(v) on the surface normal to this vector as output and so expresses a relationship between these two vectors. Because they express a relationship between vectors, tensors themselves must be independent of a particular choice of coordinate system. Taking a coordinate basis or frame of reference and applying the tensor to it results in an organized multidimensional array representing the tensor in that basis, or as it looks from that frame of reference. The coordinate independence of a tensor then takes the form of a "covariant" transformation law that relates the array computed in one coordinate system to that computed in another one. This transformation law is considered to be built in to the notion of a tensor in a geometrical or physical setting, and the precise form of the transformation law determines the type (or valence) of the tensor.

Tensors were first conceived by Tullio Levi-Civita and Gregorio Ricci-Curbastro, who continued the earlier work of Bernhard Riemann and Elwin Bruno Christoffel and others, as part of the absolute differential calculus. The concept enabled an alternative formulation of the intrinsic differential geometry of a manifold in the form of the Riemann curvature tensor. For other uses, see Tensor (disambiguation).

Note that in common usage, the term tensor is also used to refer to a tensor field. See also Glossary of tensor theory.
The stress tensor, a second-order tensor. The tensor's components, in a three-dimensional Cartesian coordinate system, form the matrix:

$$\sigma = \begin{bmatrix} T(e_1) & T(e_2) & T(e_3) \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

whose columns are the stresses (forces per unit area) acting on the $e_1$, $e_2$, and $e_3$ faces of the cube.