\[ \hat{\mathbf{n}} = \frac{(\mathbf{r} - \mathbf{r}_0) \cdot \hat{\mathbf{n}}}{|\mathbf{r} - \mathbf{r}_0|^2} \]


\[ \mathcal{L} = \sum \left( \frac{(\mathbf{r} - \mathbf{r}_0) \cdot \hat{\mathbf{n}}}{|\mathbf{r} - \mathbf{r}_0|^2} \right) \]  

project onto \( xz \) plane:

\[ \hat{\mathbf{n}}_{xz} = \hat{\mathbf{y}} \]

\[ Y = \frac{2a}{2a^2} \]

intersection:

\[ \hat{\mathbf{n}}_s (\mathbf{r} - \mathbf{r}_0) = 0 \quad (\text{normal is } \perp \text{ tangent}) \]

\[ \hat{\mathbf{n}} \cdot (r - 20a \mathbf{y}) = r - 20a \hat{\mathbf{y}} \cdot \mathbf{y} = r - 20a (\mathbf{y} \cdot \mathbf{y}) = 0 \Rightarrow r^2 - 20a = 0 \]

but \( r \) must be on the sphere: \( a^2 - 20a = 0 \Rightarrow Y = \frac{2a}{20} \)

project:

\[ x^2 + y^2 + z^2 = a^2 \quad (\text{sphere}) \]

\[ \mathcal{L} = \sum \int \frac{[r - 20a \mathbf{y} \cdot \mathbf{r}]}{[x^2 + (y - 20a)^2 + z^2]^{3/2}} \, dx \]

\[ \mathcal{L} = \sum \left( \int_{-a}^{a} \frac{[r - 20a \mathbf{y} \cdot \mathbf{r}]}{[x^2 + (y - 20a)^2 + z^2]^{3/2}} \, dx \right) \]

\[ + b - b \]

\[ \mathcal{L} = \int \left( \int_{-b}^{b} \frac{[r - 20a \mathbf{y} \cdot \mathbf{r}]}{[a^2 - 20a \sqrt{a^2 - x^2} + z^2]} \, dz \right) \]  

\[ \mathcal{L} = \int \left( \int_{-b}^{b} \frac{[a^2 - 20a \sqrt{a^2 - x^2} + z^2]}{[a^2 - 40a \sqrt{a^2 - x^2} + (20a)^2 + (20a)^2]^{3/2}} \, dz \right) \]