Problem: Show that if $AB$ is invertible, then $B$ is invertible.

Solution:

Since $AB$ is invertible, let $C = (AB)^{-1}A$. Then

$$CB = (AB)^{-1}AB = I,$$

so $C$ is a left inverse for $B$. It follows that $B$ is invertible, as claimed.