The Dynamic Absorber with Damping – AKA: Tuned Mass Damper

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February 23, 2004
Revised September 19, 2012

Figure 1. Dynamic absorber model with damping.

EOM:

\[ M_1 \ddot{x}_1 = f(t) + C_2 (\dot{x}_2 - \dot{x}_1) + K_2 (x_2 - x_1) - K_1 x_1 - C_1 \dot{x}_1 \]

or

\[ M_1 \ddot{x}_1 + (C_1 + C_2) \dot{x}_1 + (K_1 + K_2) x_1 - C_2 \dot{x}_2 - K_2 x_2 + f(t) \]  \hspace{1cm} (1)

\[ M_2 \ddot{x}_2 = -C_2 (\dot{x}_2 - \dot{x}_1) - K_2 (x_2 - x_1) \]

\[ M_2 \ddot{x}_2 - C_2 \dot{x}_1 - K_2 x_1 + C_2 \dot{x}_2 + K_2 x_2 = 0 \]  \hspace{1cm} (2)

Assuming \( f(t) = F_0 \sin \Omega t = F_0 \Im \{e^{j\Omega t}\} \) and thus assuming \( \mathbf{x}(t) = \mathbf{X} \Im \{e^{j\Omega t}\} \), where,

\[ \Im \{e^{j\Omega t}\} = \sin \Omega t \]  \hspace{1cm} (3)

denotes the \textit{Imaginary part} of \( e^{j\Omega t} \), we have

\[ \begin{bmatrix} K_1 + K_2 - M_1 \Omega^2 + j(C_1 + C_2)\Omega & -jC_2\Omega - K_2 \\ -jC_2\Omega - K_2 & K_2 - M_2 \Omega^2 + jC_2\Omega \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix} \]  \hspace{1cm} (4)

By Cramer’s rule

\[ X_1 = \frac{\det \begin{bmatrix} F_0 & -jC_2\Omega - K_2 \\ 0 & K_2 - M_2 \Omega^2 + jC_2\Omega \end{bmatrix}}{\det \begin{bmatrix} K_1 + K_2 + j(C_1 + C_2)\Omega - M_1 \Omega^2 & -(jC_2\Omega + K_2) \\ -(jC_2\Omega + K_2) & jC_2\Omega - M_2 \Omega^2 \end{bmatrix}} \]  \hspace{1cm} (5)

\[ = \frac{(K_2 - M_2 \Omega^2 + jC_2\Omega)F_0}{\triangle} \]  \hspace{1cm} (6)
where

$$\begin{align*}
\Delta &= M_1 M_2 \Omega^4 - [C_1 C_2 + (K_1 + K_2)M_2 + K_2 M_1] \Omega^2 + K_1 K_2 \\
&+ j [(K_1 C_2 + K_2 C_1) \Omega - (M_1 C_2 + (C_1 + C_2)M_2) \Omega^3] \\
&= \Re \{\Delta\} + j \Im \{\Delta\}
\end{align*}$$

(7)

Hence, the magnitude of $X_1$ is given by

$$|X_1| = F_0 \sqrt{\frac{(K_2 - M_2 \Omega^2)^2 + C_2^2 \Omega^2}{\Re^2 \{\Delta\} + \Im^2 \{\Delta\}}}$$

(8)

where

$$\Re \{\Delta\} = M_1 M_2 \Omega^4 - [C_1 C_2 + (K_1 + K_2)M_2 + K_2 M_1] \Omega^2 + K_1 K_2$$

(9)

and

$$\Im \{\Delta\} = (K_1 C_2 + K_2 C_1) \Omega - (M_1 C_2 + (C_1 + C_2)M_2) \Omega^3$$

(10)

denote the Real and Imaginary parts of $\Delta$ respectively.

**Vibration Absorber Numerical Example:**

$M_1 = 10$ kg, $K_1 = 3,947,842$ N/m, $C_2 = C_1 = 3,948$ N-s/m

$M_2 = 2$ kg, $K_2 = 7,895,681$ N/m,

$F_0 = 3,948$ N

Natural frequency before absorber: $f_n = 100$ Hz

Operating frequency: $100$ Hz

Resonance frequencies after absorber: $f_1 = 80$ Hz, $f_2 = 124$ Hz

% reduction at $100$ Hz: $99$

Based on Equation (8), it would seem that the amplitude of $x_1(t)$ is mostly a function of the ratio of $K_2$ to $M_2$ and $C_2$. If that were the case, one could pick a tiny absorber to put on a large machine or even a building as long as the correct absorber stiffness to mass ratio were correct, and fully expect it to absorb all of the vibrational energy of the original structure! Alas, life is not so simple... The absorber

![Dynamic Absorber Example: Before and After Addition of Dynamic Absorber](image.png)

Figure 2. Amplitude of $x_1(f)$ before and after addition of vibration absorber.

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Figure 3. Amplitude frequency response of the original mass ($X_1$) and tuned mass damper ($X_2$).

Figure 4. Phase frequency response of the original mass ($X_1$) and tuned mass damper ($X_2$).
mass and stiffness must be reasonably large in order to separate the resulting two modes sufficiently, but more importantly, they must be chosen large enough to absorb an appreciable amount of the kinetic and potential energy of the original structure.

The resulting amplitude and phase of the original mass ($X_1$) and tuned mass damper ($X_2$) in the combined structure are shown in Figures 3 and 4 respectively. Note that while the tuned mass damper is in phase with the forcing at 100 Hz, the original mass is about 90 degrees out of phase.