CS128 FS2013 Exam 2

This is a closed-book, closed-notes exam. The only items you are allowed to use are writing implements. Mark each sheet of paper you use with your name and the string “cs128fs2013 exam2”. If you are caught cheating, you will receive a zero grade for this exam. The max number of points per question is indicated in square brackets after each question. The sum of the max points for all the questions is 63, but note that the max exam score will be capped at 60 (i.e., there are 3 bonus points but you can’t score more than 100%). You have exactly 75 minutes to complete this exam. Keep your answers clear and concise while complete. In order to qualify for partial credit, you need to show your work. Good luck!

1. Is \{\{a, b, c\}, \{d, e\}, \{f\}\} a partition of \{a, b, c, d, e, f\}? Justify your answer! [1]

2. Let the universal set be \(U = \{a, b, c, d, e, f\}\), \(A = \{a, c, e\}\), \(B = \{b, d, f\}\), \(C = \{a, f\}\), and \(D = \{d, e\}\). Give \(\mathcal{P}((A - C) \times (B \cap D))\) where \(\mathcal{P}\) is the power set. [6]

3. Let \(\forall i \in \mathbb{Z}^+, A_i = \{x \in \mathbb{R}|1 - \frac{1}{i} < x \leq 2 + \frac{1}{i}\} = (1 - \frac{1}{i}, 2 + \frac{1}{i}]\).

   (a) Give \(\bigcup_{i=1}^{\infty} A_i\). [3]

   (b) Give \(\bigcap_{i=1}^{\infty} A_i\). [3]

4. Write the sum \(\sum_{k=0}^{n-1} (n \cdot 2^k) + \sum_{k=1}^{2^n} 2k\) in closed form and simplify as much as possible. [5]

5. Define \(f : \mathbb{N} \rightarrow \mathbb{N}\) by the rule:

   \[
   f(x) = \begin{cases} 
   x/2 & : \text{if } x \text{ is even} \\
   x + 1 & : \text{if } x \text{ is odd}
   \end{cases}
   \]

   (a) Is \(f\) a function? Prove or give a counterexample. [3]

   (b) Is \(f\) one-to-one? Prove or give a counterexample. [4]

   (c) Is \(f\) onto? Prove or give a counterexample. [4]

   (d) Is \(f\) a one-to-one correspondence (also known as bijection)? Prove or give a counterexample. [2]

   (e) Give the inverse function of \(f\) if it exists; otherwise explain why it doesn’t exist. [2]

6. Prove by mathematical induction that \(\forall n, x \in \mathbb{Z} \land n \geq 2 \land x \geq 1, (1 + x)^n > 1 + x^n\). [15]

7. Prove by mathematical induction that \(\forall n \in \mathbb{Z} \land n \geq 0, 3(n^3 + 2n)\). [15]