Lecture 3: Vectors

- Definition
- Graphical addition and subtraction of vectors
- Unit vector notation
- Vector components, magnitude and direction
- Addition and subtraction of vectors in unit vector notation
A vector is a quantity that has size (magnitude) and direction. It can be symbolized by an arrow.

Length of the arrow represents magnitude

Notation convention:
\( \hat{A} \) denotes vector of magnitude \( A = |\hat{A}| \)

*Sometimes bold-face type also indicates a vector – hard to do in handwriting*
Vector addition - graphically

\[ \vec{C} = \vec{A} + \vec{B} \]
Vector subtraction - graphically

\[ \vec{D} = \vec{A} - \vec{B} = \vec{A} + (\vec{-B}) \]
Unit vectors
Unit vector notation

\[ \vec{A} = A_x \hat{i} + A_y \hat{j} \]
Vector components

\[ A_x = +A \cos \theta \]
\[ A_y = +A \sin \theta \]

\[ \vec{A} = A \cos \theta \hat{i} + A \sin \theta \hat{j} \]
Vector components

\[ B_x = -B \sin \varphi = B \cos \Theta \]

\[ B_y = +B \cos \varphi = B \sin \Theta \]
Unit vector notation

\[ \vec{C} = C_x \hat{i} + C_y \hat{j} \]

$C_y$ is negative
Magnitude and direction

\[ A = \sqrt{A_x^2 + A_y^2} \]

\[ \tan \theta = \left| \frac{A_x}{A_y} \right| \]

\[ \tan \phi = \left| \frac{A_y}{A_x} \right| \]
A displacement of 5 km is directed $\theta = 53^\circ$ East of South. What is the displacement vector in unit-vector notation?
Example

\[ \vec{D} = D_x \uparrow + D_y \uparrow \]

\[ \theta = 53^\circ \]

\[ D = 5 \text{ km} \]
Example

\[ D_x = +D \sin \theta = +5 \text{km} (0.8) = +4 \text{km} \]
\[ D_y = -D \cos \theta = -5 \text{km} (0.6) = -3 \text{km} \]
\[ D = 5 \text{km} \]

\[ D = +4 \text{km} \uparrow + (-3 \text{km}) \uparrow \]
Vector addition in components

\[ \vec{A} = A_x \hat{i} + A_y \hat{j} \]
\[ \vec{B} = B_x \hat{i} + B_y \hat{j} \]

\[ \vec{C} = (A_x \hat{i} + A_y \hat{j}) + (B_x \hat{i} + B_y \hat{j}) \]

\[ \vec{C} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j} \]
\[ C_x = A_x + B_x \]
\[ C_y = A_y + B_y \]
Vector subtraction in components

\(\vec{A} = A_x \hat{i} + A_y \hat{j}\)
\(\vec{B} = B_x \hat{i} + B_y \hat{j}\)
\(\vec{D} = \vec{A} - \vec{B}\)

\(\vec{D} = (A_x \hat{i} + A_y \hat{j}) - (B_x \hat{i} + B_y \hat{j})\)

\(\vec{D} = (A_x - B_x) \hat{i} + (A_y - B_y) \hat{j}\)
\(D_x = A_x - B_x\)
\(D_y = A_y - B_y\)
Example

Express the vectors $\vec{C} = \vec{A} + \vec{B}$ and $\vec{D} = \vec{B} - \vec{A}$ in unit vector notation in terms of $A, B,$ and $\theta$. 
Example with tilted coordinate system
\[ N_x = 0 \]
\[ N_y = N \]

\[ P_x = -P \cos \theta \]
\[ P_y = -P \sin \theta \]

\[ W_x = W \sin \theta \]
\[ W_y = -W \cos \theta \]