Lecture 4: Motion in two dimensions

- Position, velocity, and acceleration in 2-d
- Separation of motion in x-and y-direction
- Equations for 2-d kinematics at constant acceleration
- Projectile motion
Velocity

Position vector \( \vec{r} = r_x \hat{i} + r_y \hat{j} = x \hat{i} + y \hat{j} \)

Average velocity \( \vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t} \)

Instantaneous velocity: \( \vec{v} = \frac{d\vec{r}}{dt} \)

\[ \vec{v} = v_x \hat{i} + v_y \hat{j} = \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} \]

Small change of position vector in the direction of velocity vector

\[ \vec{r}_f = \vec{r}_i + \vec{v}dt \]
Acceleration

Particle has velocity vector \( \vec{v} = v_x \hat{i} + v_y \hat{j} = \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} \)

Acceleration: \( \vec{a} = \frac{d\vec{v}}{dt} \)

\( \vec{a} = a_x \hat{i} + a_y \hat{j} = \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j} \)

\[
\begin{align*}
a_x & = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} \\
,\quad a_y & = \frac{dv_y}{dt} = \frac{d^2y}{dt^2}
\end{align*}
\]

Small change in velocity vector occurs in the direction of the acceleration vector

\( \vec{v}_f = \vec{v}_i + \vec{a} \, dt \)

Acceleration changes velocity, i.e. speed and direction of motion.
Effect of acceleration components

Components of acceleration parallel and perpendicular to velocity have different effects.

\[ d\vec{v} = \vec{a} \, dt \]

\( a_{\parallel} \) causes change in magnitude of velocity vector (speed)
\( a_{\perp} \) causes change in direction
Demonstrations

- Vertical launch of ball from traveling car
- Simultaneously dropped and horizontally launched balls
Kinematics equations

For constant acceleration:

\[
x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2 \\
y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2
\]

\[
v_x = v_{0x} + a_xt \\
v_y = v_{0y} + a_yt
\]

\[
v_x^2 = v_{0x}^2 + 2a_x(x - x_0) \\
v_y^2 = v_{0y}^2 + 2a_y(y - y_0)
\]
If only gravity acts on an object (free fall), then acceleration is a constant vector of magnitude $g$, directed down.

Effect on velocity:

$$v_x = v_{0x} + a_x t = v_{0x}$$
$$v_y = v_{0y} + a_y t = v_{0y} - gt$$

$\text{NOT Starting equations}$
Projectile motion: Simulation

http://www.walter-fendt.de/ph14e/projectile.htm
Free-fall trajectory

\[ x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \]
\[ y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \]

Worked out on the board…
A man is stranded between a river and a high vertical cliff. To get help, he wants to throw a bottle containing a message over the river. If he throws the bottle with an initial velocity $V_0$ and at a positive angle $\theta$ with respect to the horizontal, what is the minimum height $H$ he has to climb up the cliff to ensure that the bottle just barely reaches the opposite river bank, a distance $D$ away?
*You will work this out in the Special Homework.
Hint: the angle $\theta$ between initial velocity and horizontal is not given, but knowing D and H will enable you to find $\sin \theta$ and $\cos \theta$. 