Lecture 6: Newton’s Third Law

- Newton’s 3rd Law
- Action-reaction pairs
- Inclined coordinate system
- Massless ropes and massless, frictionless pulleys
- Coupled objects
Newton’s 3\textsuperscript{rd} Law

Common version:
For every action there is an equal and opposite reaction. *

*This is problematic because it suggests that first there is an action and then there is a reaction.

Better version:

If an object A exerts a force $\vec{F}_{AB}$ on object B, then object B exerts a force $\vec{F}_{BA}$ on object A, with

$$\vec{F}_{BA} = -\vec{F}_{AB}$$

equal in magnitude, opposite in direction.
Action-reaction pairs

\[ \sum F_x = M_A \, a_x \]

\[ P - F_{B \text{on} A} = M_A \, a_x \]

\[ \sum F_x = M_B \, a_x \]

\[ F_{A \text{on} B} = M_B \, a_x \]

\[ F_{B \text{on} A} = \frac{P}{M_B} \]

\[ P = (M_A + M_B) \, a_x \]

Action-reaction pair

\[ F_{A \text{on} B} = - F_{B \text{on} A} \]
Example: object at rest on a table

\[ \vec{N} = -\vec{W} \]

because \( \ddot{a} = 0 \) (Newton’s 2\textsuperscript{nd} law)

But \( \vec{N} \) and \( \vec{W} \) are not an action-reaction pair!

Action-reaction pairs are:

\( \{\vec{W} \text{ and } \vec{F}_{\text{grav Earth by } M}\} \)

and

\( \{\vec{N} \text{ and } \vec{F}_{\text{table by } M}\} \)

Forces of action-reaction pair act on two different objects
Object on inclined plane

Normal = perpendicular
Normal force must be perpendicular to surface

Weight: vertically down
Object on inclined plane

Choose axis in direction of acceleration.

Draw components of weight vector

Identify $\theta$ in the weight triangle
Components of weight vector

In this coordinate system:

\[ W_x = W \cos \theta = Mg \cos \theta \]
\[ W_y = -W \sin \theta = -Mg \sin \theta \]

CAUTION: Do not memorize!
If \( \alpha \) were angle with horizontal and if x-axis had opposite direction:

\[ W_x = -W \sin \alpha = -Mg \sin \alpha \]
\[ W_y = -W \cos \alpha = -Mg \cos \alpha \]
Coupled objects: ropes and pulleys

We make the following approximations:

- **massless, un-stretchable rope**
  → tension is constant throughout the rope

- **massless, frictionless pulley**
  → tension remains constant as rope passes over pulley

**Caution:**
If mass and spatial extension of the pulley are taken into account, the tension does **not** remain constant! We will study this with Rotational Motion in lectures 18-21.
Example with coupled objects

Two blocks are connected by a massless string. A block of mass \( m \) is on a frictionless inclined plane that makes angle \( \theta \) with the vertical, while a block of mass \( M \) hangs over a massless and frictionless pulley. Derive an expression for the acceleration of the blocks in terms of relevant system parameters.