Lecture 11: Potential energy

- Conservative and non-conservative forces
- Potential energy
- Total mechanical energy
- Energy conservation
A force is called **conservative** if the work it does on an object as the object goes between two points is **independent of the path**.

→ The work done by a conservative force along any two paths between the same two points is **the same**.
Example: Work done by gravity

\[ W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}_{Grav} \cdot d\vec{r} = -mg\hat{j} \cdot (\vec{r}_f - \vec{r}_i) \]

\[ W = -mg(y_f - y_i) \]

Depends only on \( y_i \) and \( y_f \), not on path

\[ \rightarrow \text{force of gravity is conservative} \]
Properties of conservative forces:

Reverse path, get negative:

\[ W_{A \rightarrow B} = -W_{B \rightarrow A} \]

Work done over a closed path is zero:

\[ W_{A \rightarrow A} = \oint_A \vec{F} \cdot d\vec{r} = 0 \]
Constant forces are conservative

\[ W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r} = \vec{F} \cdot \int_{\vec{r}_i}^{\vec{r}_f} d\vec{r} = \vec{F} \cdot (\vec{r}_f - \vec{r}_i) = \vec{F} \cdot \vec{D} \]

\[ \vec{D} = \vec{r}_f - \vec{r}_i \] depends only on initial and final position, not path.

Caution:
1. Force must be constant in magnitude and direction.
2. Not every conservative force has to be constant.
If the work done by a force depends on the path, the force is **non-conservative**.

Different paths between initial and final point give different amount of work.

Work for closed path is not zero.

**HW:** examine frictional force
Potential energy difference: definition

Work of conservative force $\vec{F}$ depends only on initial and final position, not on path → each pair of points has unique value of $W$ between them

Define: **Difference in potential energy** of force $\vec{F}$ between positions $\vec{r}_A$ and $\vec{r}_B$

$$\Delta U_{A\rightarrow B} = U(\vec{r}_B) - U(\vec{r}_A) = -W_{A\rightarrow B} = - \int_{\vec{r}_A}^{\vec{r}_B} \vec{F} \cdot d\vec{r}$$
Potential energy: reference point

\[ \Delta U_{A \rightarrow B} = U(\vec{r}_B) - U(\vec{r}_A) = U_B - U_A = -W_{A \rightarrow B} \]

Only differences in potential are meaningful

→ Choose arbitrary reference point \( \vec{r}_0 \) and assign it a value of potential energy \( U_0 \) that is convenient

\[ U(\vec{r}) - U(\vec{r}_0) = -W_{\vec{r}_0 \rightarrow \vec{r}} \]

\[ U(\vec{r}) = U(\vec{r}_0) - W_{\vec{r}_0 \rightarrow \vec{r}} \]
Potential energy of gravity*

\[ W_{grav} = -mg(y_f - y_i) \quad \text{(y-axis vertically up)} \]

\[ U_{grav}(\vec{r}) - U_{grav}(\vec{r}_0) = -W_{grav_{y_0}} = -[-mg(y - y_0)] \]

Choose \( \vec{r}_0 = 0 \) and assign \( U(\vec{r}_0) = 0 \)

\[ U_{grav}(\vec{r}) = mgy \quad \text{with y-axis up} \]

*near Earth’s surface
\[ y_f - y_i = 1 \text{m} \]
Potential energy of spring force

From lecture 10:

\[ W_S = \frac{1}{2} k (x_i^2 - x_f^2) \quad x = l - l_{eq} \]

\[ \Delta U_s = -W_s = -\frac{1}{2} k (x_i^2 - x_f^2) = \frac{1}{2} k (x_f^2 - x_i^2) \]

Choose \( x = 0 \) as reference point, assign \( U_s(x = 0) = 0 \)

\[ U_{spring} = \frac{1}{2} k x^2 \]
Total mechanical energy

\[ \Delta K = W_{net} = W_{conservative} + W_{other} \]

\[ K_f - K_i + (-W_{cons}) = W_{other} \]

\[ K_f - K_i + (U_f - U_i) = W_{other} \]

\[ (K_f + U_f) - (K_i + U_i) = W_{other} \]

Total mechanical energy of a system: \( E = K + U \)

\[ E_f - E_i = W_{other} \]
\[ E = K + U \]

\[ E_f - E_i = W_{other} \]

If only conservative forces act: \( W_{other} = 0 \)

\[ E_f = E_i \]

Total mechanical energy is conserved.
In a new Olympic discipline, a ski jumper of mass $M$ is launched by means of a compressed spring of spring constant $k$. At the top of a frictionless ski jump at height $H$ above the ground, he is pushed against the spring, compressing it a distance $L$. When he is released from rest, the spring pushes him so he leaves the lower end of the ski jump with a speed $V$ at a positive angle $\theta$ with respect to the horizontal.

Determine the height $D$ of the end of the ski jump in terms of given system parameters.
Tension in coupled objects

Net work done by tension in coupled system is zero

\[ W_t = \vec{t} \cdot \vec{D} = tD \]
\[ W_T = \vec{T} \cdot \vec{D} = -TD \]
\[ W_t + W_T = 0 \]
A block of mass $m$ is on a frictionless incline that makes an angle $\theta$ with the vertical. A light string attaches it to another block of mass $M$ that hangs over a massless frictionless pulley. The blocks are then released from rest, and the block of mass $M$ descends. What is the blocks’ speed after they move a distance $D$?