Lecture 20: Static Equilibrium

- Conditions for static equilibrium
- Examples
Conditions for static equilibrium

No linear acceleration:
\[ \sum \vec{F} = 0 \]

No angular acceleration:
\[ \sum \vec{\tau} = 0 \]
Two-dimensional problems

All forces act in one plane, the $xy$-plane
→ all torques perpendicular to this plane, in $z$-direction

\[
\sum F_x = 0 \\
\sum F_y = 0 \\
\sum \tau_z = 0
\]
Choice of reference point for torques

Object does not rotate $\rightarrow$ may choose any point about which to calculate torques.

Reference point along the line of action of a force: moment arm is zero $\rightarrow$ no torque

Convenient choice of reference point:
• point where several forces act
• point where unknown force acts
Easy Example:

Father and son on see-saw

Father (mass $m_1$) and son (mass $m_2$) are on a see-saw, which is a beam of mass $M$ and length $L$ that is pivoted in the middle. The son sits at one end. How far from the middle does the father have to sit for the see-saw to be in equilibrium?
Example

A massless beam of length \( L \) has its lower end pivoted at \( P \) on the floor, making an angle \( \theta \) with the floor. A horizontal cable is attached from its upper end \( E \) to a point \( A \) on a nearby wall. A rope is attached at one-fourth of the way down from the beam’s upper end, and hangs vertically downward. A disgustedly cheery purple dinosaur of mass \( M \) is attached motionless to the end of the rope.

Derive an expression for the tension in the horizontal cable \( AE \).

What are the \( x \) and \( y \) components of the force exerted by the pivot on the lower end of the beam?
A uniform beam of length $L$ and weight $W$ is set upright on a rough floor which has a coefficient of static friction $\mu$ with the beam. A constant, horizontal pulling force is applied to the beam at some height above the ground. A rope which makes an angle $\theta$ with the beam is attached to the top end of the beam. The tension in the rope is $T$. The lower end of the beam is just about to slide.

Derive an expression for the height $h$ above the ground at which the pulling force is applied, in terms of relevant system parameters.