Physics 21 Special Homework #4 (3 problems)

1. You are a scientist exploring a mysterious planet. You have performed measurements and know the following things:
   - The planet has radius $d$.
   - It is orbiting his star in a circular orbit of radius $b$.
   - It takes time $t$ to revolve around itself.
   - It takes time $T$ to complete one orbit around the star.
   - The free-fall acceleration on the surface of the planet is $a$.

Derive expressions for the mass $M_P$ of the planet and for the mass $M_S$ of its star.

2. A projectile of mass $m$ is shot directly away from the surface of a planet of mass $M$ and radius $R$ at $\frac{1}{2}$ the escape speed from the planet. Derive an expression for the maximum distance from the center of the planet the projectile reaches.
3. The kings of planet A (mass $4M$, radius $2R$) and planet B (mass $M$, radius $3R$) want to meet for negotiations. The planets are a distance $10R$ from one another, center to center. For absolute fairness, the kings (who possess no physics knowledge) decide that the meeting place $P$ is to be exactly halfway between the planets. A space capsule of mass $m$ is launched from point $\times$ on the surface of planet A by means of a giant cannon, which gives it a launch speed $V_L$. It travels directly along the line that connects the centers of both planets. Ignore the orbital motion of the planets.

a) Derive an expression for the speed $V$ with which the capsule arrives at the meeting place $P$, in terms of relevant system parameters.

b) Derive an expression for the net force (magnitude and direction) experienced by the capsule when it is at point $P$.

c) At what distance from planet A is the net gravitational force zero?