Physics 6311: Statistical Mechanics - Homework 1

due date: Friday, Jan 29, 2016

Part A: Math warmup

Problem 1: Exact differentials (10 points)

a) Test whether the following differentials are exact.
\[
\begin{align*}
du_a &= (y^2 - 2x^2) \, dx + (2xy + y^2) \, dy \\
\end{align*}
\]

\[
\begin{align*}
\end{align*}
\]

b) If the differential is exact, calculate the indefinite integral.

c) Check the dependence of the integral on the path of integration by explicitly integrating both differentials from point \((x_i, y_i) = (0, 0)\) to \((x_f, y_f) = (2, 2)\) on two different path, \((0, 0) \to (2, 0) \to (2, 2)\) and \((0, 0) \to (0, 2) \to (2, 2)\). Compare the results of the two path and that of a calculation using the indefinite integral (if it exists).

Problem 2: Properties the \(\delta\) function (6 points)

Compute the following integrals by manipulating the \(\delta\) function
\[
\begin{align*}
I_a &= \int_0^\infty dx \, x \, \delta(e^x - 2) \\
I_b &= \int_{-\infty}^\infty dx \, \cos(\pi x) \, \delta(4 - x^2) \\
\end{align*}
\]

Problem 3: Gaussian integrals (8 points)

Compute the following integral in terms of \(A\) and \(B\).
\[
I = \int_{-\infty}^\infty \int_{-\infty}^\infty dx \, dy \, e^{-(x^2 + 8xy + y^2) - Ax - By}
\]

Part B: Thermodynamics

Problem 4: Equilibrium states (6 points)

Decide which of the following states is in an equilibrium state, a non-equilibrium steady state, or not a steady state. Explain your reasoning. In some cases, the state is not a true steady or equilibrium state but close to one. Discuss under what conditions it can be treated as a steady or equilibrium state.

a) a cup of hot tea, sitting on the table while cooling down

b) the wine in a bottle that is stored in a wine cellar

c) the sun
d) the atmosphere of the earth

e) electrons in the wiring of a flashlight switched off

f) electrons in the wiring of a flashlight switched on

**Problem 5: Rubber elasticity (10 points)**

The equation of state of a rubber band can be modeled by the so-called Guth-James equation

\[ F = aT \left[ \frac{L}{L_0} - \frac{L_0^2}{L^2} \right]. \]

Here \( F \) is the tension force, \( L \) is the length of the rubber band (with \( L_0 \) being the unstretched length), \( T \) is temperature, and \( a \) is a positive constant.

a) Derive an expression for the work done in stretching the rubber band isothermally from length \( L_1 \) to length \( L_2 \).

b) A particular rubber band has an unstretched length \( L_0 = 20 \text{ cm} \), and the constant \( a \) has a value of \( 1.33 \times 10^{-2} \text{ N/K} \). How much work is performed when the band is stretched from its unstretched length to a final length of 50 cm. The temperature is constant at 20°C.

c) When the rubber band is heated at fixed tension, will its length increase or decrease with temperature? Base your answer on the equation of state.