Problem 1: Quantum corrections to classical ideal gas (18 points)

Calculate the lowest order quantum corrections to the energy and pressure of the classical ideal gas (as functions of particle number and temperature) both for fermions and for bosons. (Hint: Start from the Bose and Fermi occupation numbers. In the classical (Boltzmann) limit the average occupation numbers are small compared to 1. Expand the occupation numbers about this limit. Don’t forget the corrections to $\mu$.)

Problem 2: Bose-Einstein temperature of Helium (6 points)

The density of liquid $^4$Helium is 0.145g/cm$^3$. Calculate the Bose-Einstein condensation temperature of an ideal Bose gas of the same number density and compare it with the superfluid temperature of 2.2K.

Problem 3: Generalized Bose gas (16 points)

Consider a gas of noninteracting identical bosons of spin $S$ in $d$ dimensions. The single-particle energy-momentum relation is given by $\epsilon(p) = A|p|^z$ with positive prefactor $A$ and exponent $z$.

a) Compute the density of states $g(\epsilon)$.

b) Calculate the maximum possible particle number in excited single-particle states as a function of temperature. For which values of $d$ and $z$ does the system show Bose-Einstein condensation?

c) If there is Bose-Einstein condensation, evaluate the critical temperature $T_c$

d) Find the specific heat for temperatures $T \leq T_c$.

e) Find the pressure for temperatures $T \leq T_c$. 