31. Find the echelon form of

$$A = \begin{bmatrix} 1 & 3 & 0 & 2 & 1 \\ 1 & 3 & 0 & 4 & 2 \\ 2 & 6 & 0 & 6 & 3 \end{bmatrix},$$

the basic and free variables, rank$A$, all solutions to $Ax = 0$, and all solutions to $Ax = [7 \quad 10 \quad 17]^T$.

32. Decide whether the following vectors are linearly independent or linearly dependent. For (a), (b), and (c), also draw a picture.

(a) $\begin{bmatrix} 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \end{bmatrix}$; (b) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}$; (c) $\begin{bmatrix} 7 \\ 11 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}$;

(d) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 7 \end{bmatrix}$; (e) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$; (f) $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}, \begin{bmatrix} 9 \\ 10 \\ 11 \\ 12 \end{bmatrix}$.

33. Given are three linearly independent vectors v_1, v_2, v_3. Are the following vectors linearly independent?

(a) v_1, $v_2 + v_3$, $v_1 + v_2 + v_3$;

(b) v_1, $v_1 + v_2$, $v_1 + v_2 + v_3$.

34. Find a basis and the dimension of each of the subspaces from Problem 24.

35. Find the ranks of the following matrices. Also find a basis and the dimension of the four fundamental subspaces of each of the matrices.

(a) All the matrices from Problem 27;

(b) The 4×4-matrix on Page 105 of the textbook;

(c) The 7×7-matrix on Page 477 of the textbook;

(d) $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$; (e) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 3 & 7 \end{bmatrix}$; (f) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$;

(g) $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$; (h) $A = \begin{bmatrix} 1 & 2 & 0 & 3 & 5 \\ 0 & 0 & 1 & 4 & 6 \end{bmatrix}$; (i) $A = \begin{bmatrix} 0 & 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

36. Suppose two matrices A and B satisfy $AB = 0$. Show that the column space of B is contained in the nullspace of A.

37. Read Section 2.5 in the book and work on at least nine exercises.