43. Let $u = [1 \ -1 \ 2]^T$, $v = [-2 \ -5 \ 3]^T$, and $w = [-3 \ 1 \ 2]^T$.
   (a) Determine which pairs of the vectors u, v, and w are orthogonal.
   (b) Find the orthogonal projection of v onto the line through the origin with direction u.
44. Find the lengths, the inner products, the distances and the angles between

$$\text{(a)} \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix} \text{ (also draw a picture);} \qquad \text{(b)} \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, v = \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}; \qquad \text{(c)} \begin{bmatrix} 1 \\ 4 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \\ 3 \end{bmatrix}.$$

45. Find all vectors orthogonal to $\mathcal{R}(A)$, and all vectors orthogonal to $\mathcal{N}(A)$, if

$$\text{(a) } A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 3 & 6 & 4 \end{bmatrix}; \qquad \text{(b) } A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 4 \end{bmatrix}.$$

46. Find all vectors orthogonal to

$$\text{(a) } [1 \ 1 \ 1]^T \text{ and } [1 \ -1 \ 0]^T; \qquad \text{(b) } [1 \ 1 \ 2]^T \text{ and } [1 \ 2 \ 3]^T.$$

47. Prove the following statements where $x, y, z \in \mathbb{R}^n$. Draw a picture for $n = 2$.
   (a) $\|x\| \geq 0$ and $\|x\| = 0$ iff $x = 0$.
   (b) $\|\lambda x\| = |\lambda| \|x\|$ for all $\lambda \in \mathbb{R}$.
   (c) $(x - y) \perp (x + y)$ iff $\|x\| = \|y\|$.
   (d) $(x - z) \perp (y - z)$ iff $\|x - z\|^2 + \|y - z\|^2 = \|x - y\|^2$.
   (e) $\|x + y\|^2 + \|x - y\|^2 = 2\left(\|x\|^2 + \|y\|^2\right)$.
48. Let V be a subspace of $\mathbb{R}^n$. Prove that $V^\perp$ is also a subspace of $\mathbb{R}^n$.
49. Let V and W be subspaces of $\mathbb{R}^n$. Prove that $V \perp W$ implies that $V \cap W = \{0\}$.
50. Let $P = \left\{[a \quad b \quad c]^T \in \mathbb{R}^3 : \ a + 2b - c = 6\right\}$.
   (a) Give three points that are in P and three points that are not in P. Is P a subspace of $\mathbb{R}^3$?
   (b) Find the subspace Q of $\mathbb{R}^3$ that has dimension 2 and no point in common with P. Give three points that are in Q and three points that are not in Q.
   (c) Find $Q^\perp$. What is the dimension of $Q^\perp$? Give three points that are in $Q^\perp$ and three points that are not in $Q^\perp$.
   (d) Find, if possible, matrices A and B such that $\mathcal{N}(A) = Q^\perp$ and $\mathcal{R}(B) = Q^\perp$.