

1. Determine an integrating factor for  $xy' - 3x \sin(x) + 2y - x^2 \cos(x) = 0$  and find some solution of this equation. Give this solution in explicit form.
2. We invest \$50000 in a bank where interest is compounded monthly at a rate of 6% a year. At the end of each month we withdraw \$100. Let  $y_k$  be the amount of money on the account at the beginning of the  $k$ th month.
  - (a) Find  $y_1$  and  $y_2$ .
  - (b) Find a difference equation for  $y_k$ .
  - (c) Solve the resulting initial value problem.
  - (d) Determine the amount of money in the account after ten years.
  - (e) Will the money ever be gone? If so, when?
3. Solve the initial value problem  $y'' - \frac{5}{2}y' + y = 0$ ,  $y(0) = 0$ ,  $y'(0) = 1$ .
4. Consider the equation  $y'' - y' \cos(t) - y \cosh(t) = 0$  (do not attempt to solve this equation). Let  $y_1$  and  $y_2$  be two solutions of this equation such that  $y_1(0) = 0$ ,  $y_1'(0) = 1$ , and  $y_2(0) = -2$ .
  - (a) Find the Wronskian of  $y_1$  and  $y_2$  evaluated at 0.
  - (b) Find the Wronskian of  $y_1$  and  $y_2$  for any time  $t$ .
  - (c) If  $y_1(\frac{\pi}{2}) = 2$  and  $y_1'(\frac{\pi}{2}) = 0$ , determine the value of  $y_2'(\frac{\pi}{2})$ .
5. Given are the first two members of a sequence  $y_0 = 0$  and  $y_1 = 1$ . Using two consecutive members, the next member of the sequence is constructed in the following way: You take the second one of these two numbers, multiply it by 5, divide it by 2, and then subtract the first one.
  - (a) Find  $y_2$ ,  $y_3$ , and  $y_4$  (give the decimal representations of these numbers).
  - (b) Find a difference equation that is satisfied by the members of this sequence.
  - (c) Solve the resulting initial value problem.
  - (d) Use the obtained formula to calculate  $y_5$ .
  - (e) What is the limit of  $\frac{y_{n+1}}{y_n}$  as  $n$  tends to  $\infty$ ?