10. Prove the following statements using the Principle of Mathematical Induction:

    (a) $\forall n \in \mathbb{N} \ \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$;

    (b) $\forall n \in \mathbb{N} \ \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$;

    (c) $\forall n \in \mathbb{N} \ \sum_{k=1}^{n} (-1)^{k+1} k^2 = \frac{(-1)^{n+1} n(n+1)}{2}$.

11. Let $P(n): \ \sum_{k=1}^{n} (2k) = (n+2)(n-1)$. Find the truth values of the following propositions:

    (a) $\forall n \in \mathbb{N} \ P(n) \to P(n+1)$;

    (b) $\overline{\forall n \in \mathbb{N} \ P(n)}$.

12. By experimenting with some values of n, guess a formula for the sum

$$\sum_{k=1}^{n} \frac{1}{k(k+1)},$$

    and then use mathematical induction to verify your formula.

13. Prove the following statements using the Principle of Mathematical Induction:

    (a) $\forall n \in \mathbb{N} \setminus \{1,2\} \ 2n+1 \leq 2^n$;

    (b) $\forall n \in \mathbb{N} \setminus \{1,2,3\} \ 2^n \geq n^2$;

    (c) $\forall n \in \mathbb{N} \ 3^n \geq n 2^n$;

    (d) $\forall n \in \mathbb{N} \ 5 | (11^n - 6)$;

    (e) $\forall n \in \mathbb{N} \ 4 | (6 \cdot 7^n - 2 \cdot 3^n)$;

    (f) $\forall x \geq -1 \ \forall n \in \mathbb{N} \ (1+x)^n \geq 1 + nx$.