23. Determine the order of the following ODEs, and whether they are linear or nonlinear. Also verify directly that the given function is a solution of the equation:

   (a) $y' = t\sqrt{y}$ with $y(t) = \frac{t^4}{16}$;

   (b) $y'' + 16y = 0$ with $y(t) = 5\cos(4t) + 3\sin(4t)$;

   (c) $y' = 25 + y^2$ with $y(t) = 5\tan(5t)$;

   (d) $t^2y'' - ty' + 2y = 0$ with $y(t) = t\cos(\ln(t))$;

   (e) $y' = 2\sqrt{|y|}$ with $y(t) = t|t|$.

24. Determine all values of r for which the given ODE has solutions of the form $y(t) = e^{rt}$:

   (a) $y' + 2y = 0$;

   (b) $y'' + y' - 6y = 0$;

   (c) $y''' - 3y'' + 2y' = 0$.

25. Determine all values of r for which the given ODE has solutions of the form $y(t) = t^r$, $t > 0$:

   (a) $t^2y'' + 4ty' + 2y = 0$;

   (b) $t^2y'' - 4ty' + 4y = 0$.

26. Use exactly the same steps as in Example 2.2 (vi) from the lecture to find all solutions of the ODE $y' + 4y + 2 = 0$. Also, give the solution y of this ODE that satisfies $y(1) = 2$. Finally, let t_0 and y_0 be arbitrary real numbers and find the solution y of the ODE that satisfies $y(t_0) = y_0$.

27. Let $N(t)$ be the number of atoms of a radioactive element at time t. We assume that the element disintegrates at a rate proportional to the amount present.

   (a) Find a differential equation for N;

   (b) Show that the half-time T, which is defined by $N(t_0 + T) = \frac{1}{2}N(t_0)$, is independent of t_0. Express this half-time as a function of only the constant of proportionality.

   (c) If the constant of proportionality is 0.03 (inverse of days), after what time will 100 mg of the radioactive material be reduced to 80 mg?

   (d) If 100 mg of the radioactive material are reduced to 80 mg in 6 days, determine the rate of proportionality and the amount of material left over after 8 days.

28. Draw a direction field for the given ODE. Based on the direction field, determine the behavior of $y(t)$ as $t \to \infty$:

   (a) $y' = -1 - 2y$;

   (b) $y' = t + 2y$.