29. Solve the following initial value problems by separating the variables. Give the solutions explicitly and find their domains.

(a) $y' = (1-2t)y^2$, $y(0) = -\frac{1}{6}$ (b) $y' = -\frac{x}{y}$, $y(1) = 1$;
(c) $y' = \frac{x^2}{y}$, $y(0) = 1$; (d) $y' = \frac{x^2}{y}$, $y(0) = -1$;
(e) $y' = \frac{3x^2-1}{3+2y}$, $y(0) = 1$; (f) $\sin(2t) + \cos(3y)y' = 0$, $y(\frac{\pi}{2}) = \frac{\pi}{3}$.

30. Consider the linear first order equation with constant coefficients $y' = ry + k$.

(a) Find the general solution.
(b) Find all constant solutions.
(c) Find the solution with $y(0) = 2$.
(d) For a given point (t_0, y_0), find the solution that goes through this point.
(e) Characterize all increasing solutions. Characterize all decreasing solutions.
(f) Determine the behavior of the solutions as $t \to \infty$.

31. Find the solutions of the following initial value problems:

(a) $y' = 5y - 1$, $y(0) = 2$;
(b) $y' = -y + 4$, $y(1) = -1$;
(c) $5y' = 2y - 3$, $y(-2) = 3$;
(d) $3y' - 2y = 1$, $y(-1) = 0$;
(e) $-2y' + 2y - 4 = 0$, $y(5) = 10$.

32. Consider a certain product on the market. Let a demand function $D(t)$ and a supply function $S(t)$ for this product be given. Also, let the function $P(t)$ describe the market price of the product (as a function of the time t). We assume that S and D depend linearly on the market price P: $D(t) = \alpha + aP(t)$, $S(t) = \beta + bP(t)$.

(a) According to the model, should we assume $a < 0$ or $a > 0$?
(b) According to the model, should we assume $b < 0$ or $b > 0$?
(c) Now we assume that P is changing proportionally to the difference $D - S$, with constant of proportionality γ. According to the model, should we assume $\gamma < 0$ or $\gamma > 0$?
(d) Derive a differential equation for P and solve it.
(e) Calculate the so-called equilibrium price of the product, i.e., determine $\lim_{t\to\infty} P(t)$.

33. Solve the following initial value problems:

(a) $y' - y = 2te^{2t}$, $y(0) = 1$; (b) $y' + 2y = te^{-2t}$, $y(1) = 0$;
(c) $ty' + 2y = t^2 - t + 1$, $y(1) = \frac{1}{2}$, $t > 0$; (d) $y' + \frac{2}{t}y = \frac{\cos(t)}{t^2}$, $y(\pi) = 0$, $t > 0$;
(e) $y' - 2y = e^{2t}$, $y(0) = 2$; (f) $ty' + 3y = t^2$, $y(1) = 0$;
(g) $y' = -t^2 y$, $y(0) = 1$; (h) $y' + 2ty = 2te^{-t^2}$, $y(2) = 0$.