1. Find the solution of the problem

$$u_t - ku_{xx} + btu = 0, \quad u(x,0) = \phi(x),$$

where $b, k > 0$. (Hint: Problems 37 and 38.)

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2. Prove that the total energy for the wave equation

$$E(t) = \frac{1}{2}\int_0^l \left\{ \frac{1}{c^2} u_t^2(x,t) + u_x^2(x,t) \right\} \mathrm{d}x$$

is conserved when having Neumann boundary conditions.

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3. Find the Fourier series of

$$f(x) = \begin{cases} -3 & \text{if } -\pi \le x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } 0 < x \le \pi. \end{cases}$$

Does the Fourier series of f converge pointwise to f in $(-\pi, \pi)$?

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4. Let $c > 0$ and $N \in \mathbb{N}$. Consider the discrete problem

$$\Delta_m u(n+1, m) = k\Delta_{nn} u(n,m), \quad u(0,m) = u(N,m) = 0$$

(with $\Delta_m u(n,m) = u(n, m+1) - u(n,m)$, $\Delta_n u(n,m) = u(n+1, m) - u(n,m)$ etc.) and find solutions by separating the variables $n \in \{0, 1, \ldots, N\}$ and $m \in \mathbb{N}_0$. (Hint: Problem 46.)