1. Let $A : \mathcal{H} \to \mathcal{K}$ be a linear operator. Let $\{x_j\}_{j\in\mathbb{N}} \subset \mathcal{H}$. Show that the following are equivalent:

   (a) $A \in \mathcal{B}(\mathcal{H}, \mathcal{K})$;

   (b) $x_j \to x \Longrightarrow Ax_j \to Ax$;

   (c) $x_j \rightharpoonup x \Longrightarrow Ax_j \rightharpoonup Ax$;

   (d) $x_j \to x \Longrightarrow Ax_j \rightharpoonup Ax$.

   (Hint for (d) $\Longrightarrow$ (a): Suppose not. Then there is a sequence $\{x_n\}$ with $\|x_n\| = 1$ such that $\|Ax_n\| \geq n^2$. Consider the sequence $\{\frac{1}{n}x_n\}$.)