

2. Show the following properties of weak and weak\* convergence in Banach spaces:

- (a) The weak limit of a sequence is unique;
- (b) the weak\* limit of a sequence is unique;
- (c) convergence implies weak convergence;
- (d) convergence implies weak\* convergence;
- (e) if  $y_k \xrightarrow{*} y$  for  $k \rightarrow \infty$ , then  $\|y\| \leq \liminf_{k \rightarrow \infty} \|y_k\|$ ;
- (f) if  $x_k \rightharpoonup x$  for  $k \rightarrow \infty$ , then  $\|x\| \leq \liminf_{k \rightarrow \infty} \|x_k\|$ ;
- (g) weakly convergent sequences are bounded;
- (h) weakly\* convergent sequences are bounded;
- (i) if  $x_j \rightarrow x$  and  $y_j \xrightarrow{*} y$ , then  $\langle x_j, y_j \rangle \rightarrow \langle x, y \rangle$  for  $j \rightarrow \infty$ ;
- (j) if  $x_j \rightharpoonup x$  and  $y_j \rightarrow y$ , then  $\langle x_j, y_j \rangle \rightarrow \langle x, y \rangle$  for  $j \rightarrow \infty$ .