1. Given are the following data:

| Ingredients (per 500ml) | Lager | Pilsener | Export | Bock |
|---|---|---|---|---|
| g carbohydrates | 20 | 17 | 20 | 30 |
| g proteine | 1.5 | 1.5 | 1.55 | 1.6 |
| calories | 185 | 190 | 250 | 310 |
| g alcohol | 18 | 20 | 21.5 | 24.5 |
| g minerals | 1.25 | 1.25 | 1.5 | 1.75 |
| costs ($/500ml) | 1.10 | 1.30 | 1.20 | 1.50 |

Write an LP that calculates the cheapest daily menu that contains at least 250g carbohydrates, 30g proteine, 1000 calories, 150g alcohol, and 25g minerals.

2. Prove that if $x$ and $y$ are feasible for an LP in standard form, then every point on the line segment $\overline{xy}$ is also feasible. Prove that if $x$ and $y$ are optimal, then every point on the line segment $\overline{xy}$ is also optimal.

3. A chemical factory has to mix four gases in such a way that
   - the total gas volume is exactly $100m^3$;
   - the mixture doesn't cost more than $500;
   - the portion of the fourth gas is no more than 50%;
   - at least $8m^3$ of the second gas is contained in the mixture;
   - at most $3g/m^3$ sulfur is in the mixture;
   - the heating value of the mixture should be as large as possible.

   The following table shows the specific data for all four gases:

| | Gas 1 | Gas 2 | Gas 3 | Gas 4 |
|---|---|---|---|---|
| sulfur contained in $g/m^3$ | 7.244 | 0 | 0.2 | 2 |
| costs in $ per $m^3$ | 15 | 38 | 1.40 | 2.90 |
| heating value per $m^3$ | 1.056 | 2.043 | 0.17 | 5.74 |

Introduce appropriate decision variables and state the problem as an LP in standard form.

4. Use the Two-Phase Simplex Method to solve

$$\begin{cases} 2x_1 + 2x_2 + 4x_3 - x_4 & \to & \min \\ x_1 + 2x_2 + 2x_4 & = & 1 \\ x_1 + x_2 + 2x_3 & = & 1 \\ x_1, x_2, x_3, x_4 & \geq & 0. \end{cases}$$

5. Work on Problem 6 a-d of Section 6.3 in the textbook.

6. Solve

$$\begin{cases} 7x_1 + 10x_2 & \to & \min \\ x_1 + x_2 & \geq & 1 \\ x_1 + 2x_2 & \geq & 3 \\ x_1, x_2 & \geq & 0 \end{cases}$$

by solving the dual LP and using the complementary conditions.

7. Solve the transportation problem given by

$$a = \begin{bmatrix} 12 \\ 8 \end{bmatrix}, b = \begin{bmatrix} 7 \\ 5 \\ 6 \\ 2 \end{bmatrix}, C = \begin{bmatrix} 3 & 2 & 5 & 0 \\ 4 & 1 & 2 & 0 \end{bmatrix}$$

where $a$ is the supply vector, $b$ is the demand vector, and $C$ is the cost matrix. Use the northwest corner method to find an initial bfs.

8. Use Dijkstra's Algorithm to work on Problem 5 of Section 8.2 in the textbook.