39. Recall the optimal solutions of the LP in Homework Problem 19 and of the dual of the LP in Homework Problem 37. Use the Dual Theorem to calculate the optimal solutions of the duals of these LPs.

40. A beer distributor has three branches in the cities A, B, and C, in which 40, 30, and 50 kegs of beer are stored. The distributor delivers, and one day three customers D, E, and F are requesting 30, 60, and 30 kegs of beer. The delivery costs per keg of beer are: \$2 from A to D, \$3 from A to E, \$2 from A to F, \$1 from B to D, \$1 from B to E, \$2 from B to F, \$3 from C to D, \$2 from C to E, \$1 from C to F. Rewrite the problem as an LP of the form

$$\begin{cases} c^T x \to \min \\ Tx = b \\ x \geq 0 \end{cases}$$

by giving the vectors c and b and the matrix T. Solve this LP using LINDO.

41. Solve the following transportation problems (manually), where a is the supply vector, b is the demand vector, and C is the cost matrix.

(i)

$$a = \begin{bmatrix} 45 \\ 40 \end{bmatrix}, b = \begin{bmatrix} 25 \\ 30 \\ 30 \end{bmatrix}, C = \begin{bmatrix} 6 & 3 & 0 \\ 9 & 5 & 0 \end{bmatrix}$$

(ii)

$$a = \begin{bmatrix} 12 \\ 8 \end{bmatrix}, b = \begin{bmatrix} 7 \\ 5 \\ 6 \\ 2 \end{bmatrix}, C = \begin{bmatrix} 3 & 2 & 5 & 0 \\ 4 & 1 & 2 & 0 \end{bmatrix}$$

(iii)

$$a = \begin{bmatrix} 5 \\ 6 \\ 5 \\ 2 \end{bmatrix}, b = \begin{bmatrix} 5 \\ 7 \\ 6 \end{bmatrix}, C = \begin{bmatrix} 6 & 3 & 7 \\ 4 & 3 & 5 \\ 9 & 10 & 11 \\ 0 & 0 & 0 \end{bmatrix}$$